POLITEKNIK MALAYSIA

KEMENTERIAN PENGAJIAN TINGGI

KOLEJ KOMUNITI MALAYSIA

MATHEMATICS FOR TECHNOLOGY

VOLUME 2

LEARNING MODULE FOR POLYTECHNIC

WRITTEN BY:
MOHD SUMAZLIN BIN MAHAMED
MOHD NAWI BIN AB. RAHMAN@ISMAIL
NURUL AMALINA BINTI IBRAHIM

MATHEMATICS FOR TECHNOLOGY
Vol. 2

Mohd Sumazlin Bin Mahammed
Mohd Nawi bin Ab. Rahman@Ismail
Nurul Amalina Binti Ibrahim

Jabatan Matematik Sains Komputer
Politeknik Jeli Kelantan
2021

MATHEMATICS FOR TECHNOLOGY

VOLUME 2

Published by:
Politeknik Jeli Kelantan,
Jalan Raya Timur Barat,
17600 Jeli
Kelantan

Email:
sumazlin@pjk.edu.my
nawi@pjk.edu.my
amalina@pjk.edu.my

Cetakan Pertama 2021

Perpustakaan Negara Malaysia
Cataloguing-in-Publication Data

Mohd Sumazlin Mahammed, 1980-
MATHEMATICS FOR TECHNOLOGY. VOLUME 2 / Mohd Sumazlin Bin Mahammed, Mohd Nawi bin Ab. Rahman@Ismail, Nurul Amalina Binti Ibrahim.
Mode of access: Internet
eISBN 978-967-2760-01-6
1. Mathematics.
2. Technology--Mathematics.
3. Government publications--Malaysia.
4. Electronic books.
I. Mohd Nawi bin Ab. Rahman@Ismail, 1974-.
II. Nurul Amalina Ibrahim, 1991-. III. Title.
510

Preface

Thankful to Allah s.w.t for His Bless along the process of completing this module. Congratulations to all lecturers who are involved in drafting, writing and editing this module namely En. Mohd Nawi Bin Ab. Rahman@Ismail, Cik Nurul Amalina Binti Ibrahiman and En. Mohd Sumazlin Bin Mohamed. We would like to express our gratitude and special thanks to our Head of Department, En. Mohd Nor bin Yusof and appreciate to all of members from Mathematics, Science & Computer Department, Politeknik Jeli Kelantan (PJK) for their support and assistance with numerous materials for this work.

Mathematics For Technology Volume 2 is designed for a typical first semester general mathematics students in Polytechnic Jeli Kelantan, incorporating innovative features to enhance student and user learning. The module guides students the core concepts of Measurements, Function & Graph and Statistics and helps them understand how those concepts apply to their lives and the world around them. With this objective in mind, the content of this module have been developed and arranged to provide a logical progression from fundamental to more advanced concepts, building upon what students have already learned and emphasizing connections between topics and between theory and application. It also provides step-by-step example followed by exercise for every subtopic. In addition, review questions are attached at the of every chapter as the enhancement for students and instructor.

Thus, we are really hope that this module will be useful for students in enhancing their mathematics skills and developing their interest towards mathematics. It also will be valuable additional reference for lecturers and others.

Thank you

Mohd. Shakirurahman Bin Ismail

Editor

TABLE OF CONTENT

CHAPTER 1

MEASUREMENT

Written By: En. Mohd Sumazlin Bin Mahamed

1.0 MEASUREMENT

1.0 Convert unit

A unit conversion expresses the same property as a different unit of measurement.

1.1.1 Length Units

The Table 3.1 can be used to convert between some common length units.

Table 1.1 : Length Units

Micrometer [μm]	Milimeter [mm]	Centimeter [cm]	Meter [m]	Kilometer [km]	Inche [in]	Feet [ft]	Yard [yd]	Mile [mi]
1	0.001	0.0001	0.000001	1.00E-09	3.94E-05	0.00000328	0.00000109	6.21E-10
1.00E+03	1	0.01	0.01	0.000001	0.03937	0.003281	0.001094	6.21E-07
1.00E+04	10	1	0.01	0.00001	0.393701	0.032808	0.010936	6.21E-06
1.00E+06	1000	100	1	0.001	39.37008	3.28084	1.093613	6.21E-04
1.00E+09	1000000	100000	1000	1	39370.08	3280.84	1093.613	6.21E-01
2.54E+04	25.4	2.54	0.0254	0.0000254	1	0.083333	0.027778	1.58E-05
3.05E+05	304.8	30.48	0.3048	0.0003048	12	1	0.333333	1.89E-04
9.14E+05	914.4	91.44	0.9144	0.0009144	36	3	1	5.68E-04
1.61E+09	1609344	160934.4	1609.344	1.609344	63360	5280	1760	1

Example 1

Convert meters to inches
3m = 3 (39.37008)
= 118.1102 in

Convert inches to feet
60 in = 60 (0.083333)
= 5 ft

Convert kilometers to miles
48.3km = 48.3 (0.621)
= 30 mi

Exercise 1.1

a. 5 meters to inches
b. 96 inches to feet
c. 120 kilometers to miles

1.1.2 Weight and Mass Units

Mass is used to measure the weight of an object.

Table 1.2 : Weight and Mass Units

miligram [mg]	gram [g]	kilogram [kg]	ounce [oz]	pound [lbs]	ton [t]
1	0.001	0.000001	3.53E-05	2.2E-06	1.00E-09
1000	1	0.001	0.035274	0.002205	0.000001
1000000	1000	1	35.27396	2.204623	0.001
28349.52	28.34952	0.02835	1	0.0625	2.83E-05
453592.4	453.5924	0.453592	16	1	0.000454
1E+09	1000000	1000	35273.96	2204.623	1

Example 1.2

Convert grams to kilograms

2000g = 2000 (0.001)
= 2kg

Convert ounces to pounds

50 oz = 50 (0.0625)
= 3.125 oz

Convert tons to kilograms

30t = 30 (1000)
= 30,000 kg

Exercise 1.2

a. 3000 grams to kilograms
b. 4500 grams to kilograms
c. 40 ounces to pounds
d. 5 tons to kilogram
e. 1200 kilograms to tons

1.1.3 Area

Area is used to measure 2-dimensional and unit of area is square units

Table 1.3 : Area Units

square centimeter [cm^2]	square meter [m^2]	square inch [in^2]	square foot [ft^2]	acre [ac]	hectare [ha]
1	0.0001	0.1550	0.001076	2.47E-8	1.E-8
10000	1	1550	10.76	0.0002471	0.0001
6.4516	0.00064516	1	0.006944	1.59E-7	6.45E-8
929	0.0929	144	1	0.00002296	0.00000929
4.E+8	4046.86	6272640	43560	1	0.4047
1.E+8	10000	15500031	107639	2.471	1

Example

Convert square meter to square inch

$2m^2 = 2(1550)$

$= 3100\ in^2$

Convert square inch to square foot

$3320\ in^2 = 3320\ (0.006944)$

$= 23\ ft^2$

Convert hectare to acre

$8\ ha = 8\ (2.471)$

$= 19.77\ ac$

Exercise 1.3

a. 3 square meter to square inch
b. 1100 square inch to square foot
c. 25 square foot to square inch
d. 2 hectares to acre
e. 5 acres to hectare

1.1.4 Volume

Volume is a three-dimensional measurement; that is, it measures the XYZ axis.

Table 1.4 : Volume Units

cubic centimeter [cm^3]	cubic meter [m^3]	cubic inch [in^3]	cubic foot [ft^3]	Liter [L]	gallon [gal] UK
1	0.000001	0.061	0.0000353	0.001	0.00022
1000000	1	61023.744	35.315	1000	219.969
16.387	0.0000163	1	0.000578	0.0163	0.0036
28316.85	0.0283	1728	1	28.317	6.23
1000	0.001	61.024	0.0353	1	0.21
4546.09	0.0045	277.42	0.16	4.546	1

Example

Convert cubic meter to liter

$5000m^3 = 5000/1000$

$= 5L$

Convert liter to cubic meter

$3L = 3(1000)$

$= 3000\ m^3$

Convert gallon to cubic meter

$600\ gal = 6000\ (0.0045)$

$= 2.7\ m^3$

Exercise 1.4

a. 8 cubic meter to liter
b. 4 liter to cubic meter
c. 2 gallon to cubic meter
d. 5 gallon to cubic inch
e. 5 gallon to liter

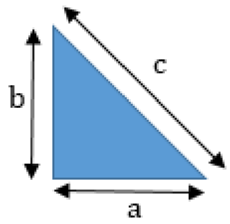
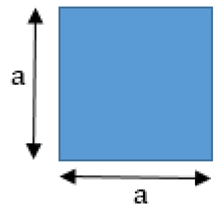
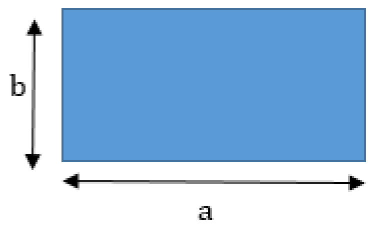
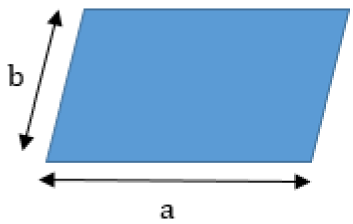
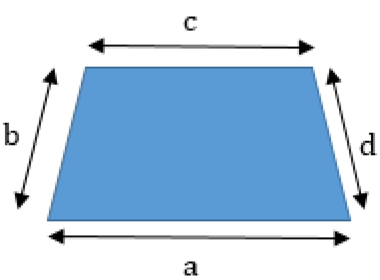
1.2 Perimeter, Surface Area and Volume

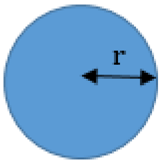
The perimeter is a path the outlines a shape.
The surface area is the sum of all the shapes that cover the surface of the object.
Volume is the space that a substance or contains.

1.2.1 Perimeter

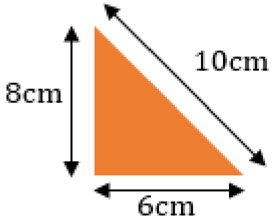
Perimeter is measured in linear units.

Table 1.5 : Perimeter Formulas

Name Shape	Diagram	Perimeter
Triangle		P = a + b + c
Square		P = 4a
Rectangle		P = 2a + 2b
Parallelogram		P = 2a + 2b
Trapezoid		P = a + b + c + d

Circle		P= $2\pi r$

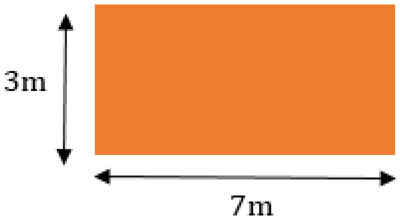
Example 1
Find the perimeter of triangle below



Solution

$P = a + b + c$
$\quad = 6 + 8 + 10$
$\quad = 24$ cm

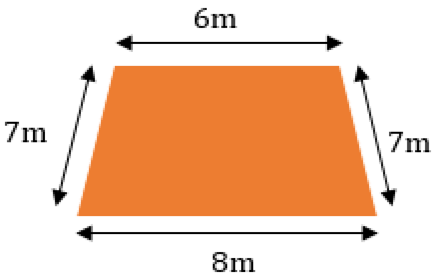
Example 2
Find the perimeter of rectangle below



Solution

$P = 2a + 2b$
$\quad = 2(7) + 2(3)$
$\quad = 14 + 6$
$\quad = 20$ m

Example 3
Find the perimeter of trapezoid below

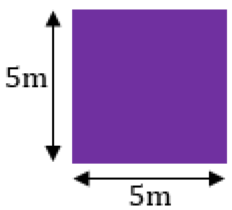


Solution

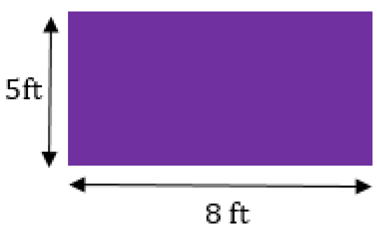
$P = a + b + c + d$
$\quad = 8 + 7 + 6 + 6$
$\quad = 27\text{ m}$

Exercise 1.5
Find the perimeter of each figure given below

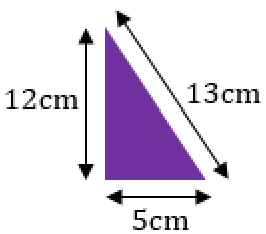
a.



b.

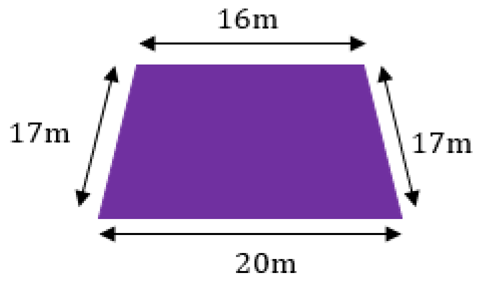


c.

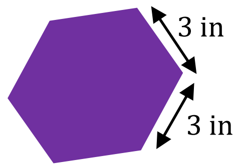


.

d.



e.

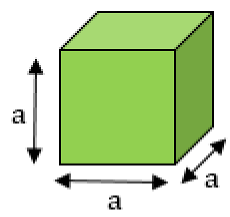
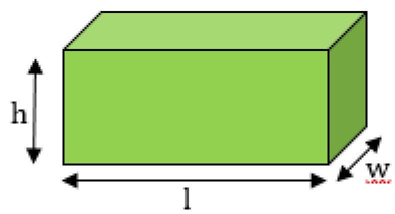


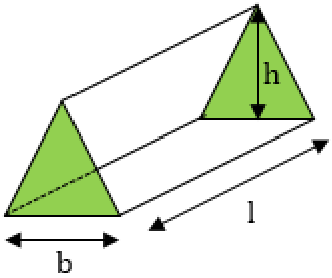
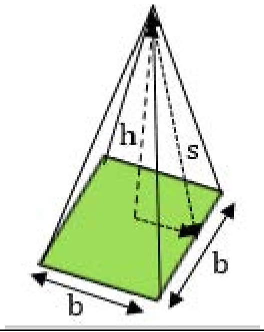
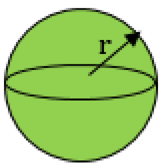
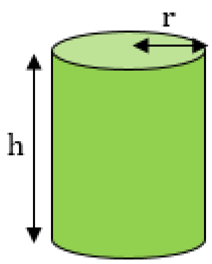
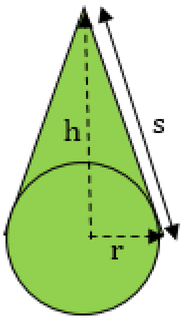
1.2.2 Surface Area and Volume

The surface area is the sum of all the areas of all the shapes that cover the surface of the object. The International System of Units (SI) for surface area is the square meter, or m^2.

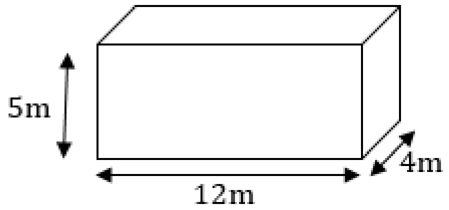
Volume is the quantification of the three-dimensional space a substance. The SI unit for volume is the cubic meter, or m^3.

Table 3.6 : Surface Area and Volume Formulas

Name Shape	Diagram	Surface Area	Volume
Cube		$SA = 6a^2$	$V = a^3$
Rectangle		$SA = 2hl + 2hw + 2lw$	$V = whl$

Prism		$SA = bh + lb + 2ls$	$V = \frac{1}{2} bhl$
Pyramid		$P = b^2 + 2bs$	$V = \frac{1}{3} s^2h$
Sphere		$SA = 4\pi r$	$V = \frac{4}{3} \pi r^2$
Cylinder		$SA = 2\pi r^2 + 2\pi rh$	$V = 2\pi r^2h$
Cone		$SA = \pi r^2 + \pi rs$	$V = \frac{1}{3} \pi r^2h$

Example 1
Figure below shows a rectangle. Calculate the surface area and volume.



Solution

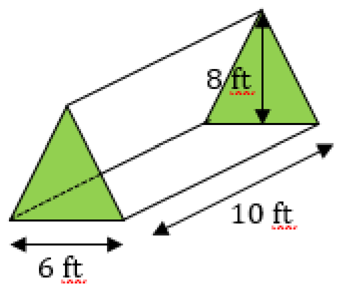
Surface Area

SA = 2hl + 2hw + 2lw
= 2(5)(12) + 2(5)(4) + 2(12)(4)
= 120 + 40 + 96
= 256 m^2

Volume

V = whl
= (4)(5)(12)
= 240 m^3

Example 2
Figure shows a prism with 8 feet height. Calculate the surface area and volume.



Solution

Surface Area

SA = bh + lb + 2ls
= (6)(8) + (10)(6) + 2(10)(8.54)
= 48 + 60 + 170.80
= 278.80 ft^2

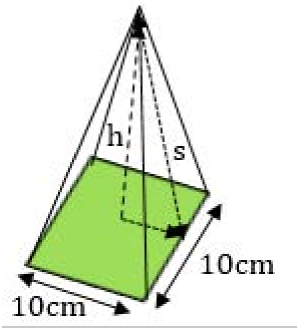
$S = \sqrt{3^2 + 8^2}$
$= \sqrt{73}$
= 8.54

Volume

$V = \frac{1}{2}bhl$
$= \frac{1}{2}(6)(8)(10)$
= 240 ft^3

Example 3

Figure below shows a pyramid with 20cm meters height. Calculate the surface area and volume.



Solution

$s = \sqrt{20^2 + 5^2}$

$= \sqrt{425}$

$= 20.6$ cm

The triangle

$= \frac{1}{2}sh$

$= \frac{1}{2}(20.6)(10)$

$= 103\ cm^2$

<u>Surface Area</u>

$SA = 10^2 + 2(10)(20.6)$

$= 100 + 412$

$= 512\ cm^2$

<u>Volume</u>

$V = \frac{1}{3}s^2h$

$= \frac{1}{3}(20.6)^2(20)$

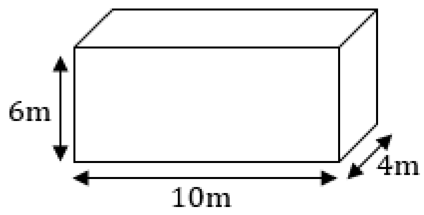
$= \frac{1}{3}(8487.20)$

$= 2829\ cm^3$

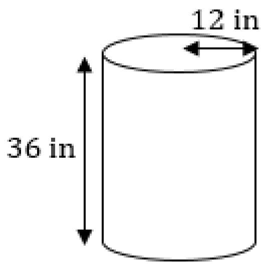
Exercise 1.6

Calculate the surface area and volume figure below

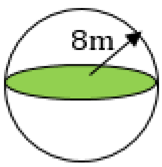
a.



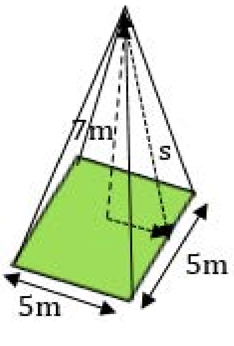
b.



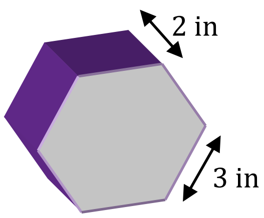
c.



d.



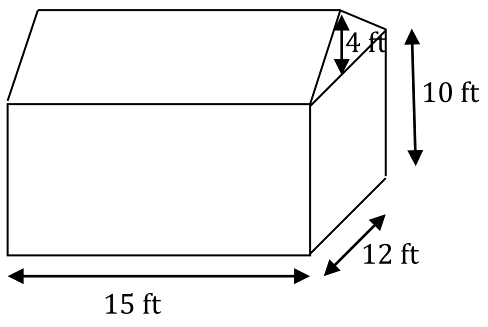
e.



1.3 Surface Area and Volume of Two or Three Shapes

Example 1

Refer to the figure below, calculate the surface area and volume.



Solution

Surface Area

$SA_{rect} = 2hl + 2hw + 2lw$
$= 2(10)(15) + 2(10)(12) + 2(15)(12)$
$= 300 + 240 + 360$
$= 900\ ft^2$

$SA_{pri} = bh + lb + 2ls$
$= (12)(4) + (15)(12) + 2(15)(7.21)$
$= 48 + 180 + 216$
$= 444\ ft^2$

$s = \sqrt{4^2 + 6^2}$
$= \sqrt{52}$
$= 7.21\ ft$

$SA_T = SA_{rect} + SA_{pri}$
$= 900 + 444$
$= 1344\ ft^2$

Volume

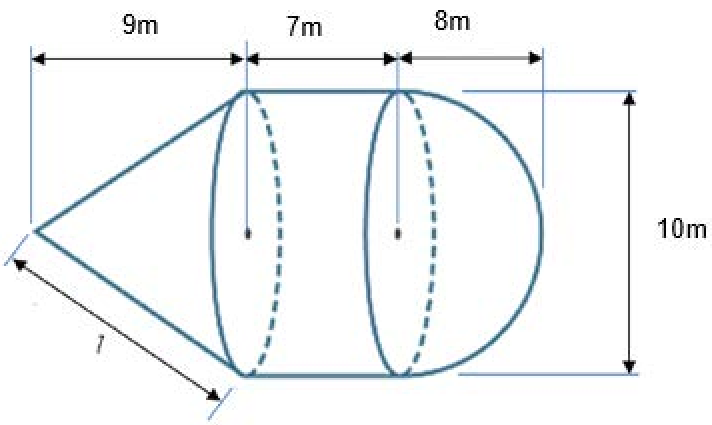
$V_{rect} = whl$
$= (12)(10)(15)$
$= 1800\ ft^3$

$Vpri = \frac{1}{2}bhl$
$= \frac{1}{2}(12)(10)(15)$
$= 900\ ft^3$

$V_T = 1800 + 900$
$= 2700\ ft^3$

Example 2

Refer to the figure below, calculate the surface area and volume.



Solution

Surface Area

$SA_{hs} = \frac{1}{2}(4\pi r^2)$
$= 2\pi r^2$
$= 2\pi(5)^2$
$= 157.08 \text{ m}^2$

$SA_{cyc} = 2\pi r^2 + 2\pi rh$
$= 2\pi(5)^2 + 2\pi(5)(7)$
$= 157.08 + 219.91$
$= 371.99 \text{ m}^2$

$SA_{cone} = \pi r^2 + \pi rs$
$= \pi(5)^2 + \pi(5)(10.30)$
$= 78.54 + 161.79$
$= 240.33$

$s = \ell$
$= \sqrt{5^2 + 9^2}$
$= \sqrt{106}$
$= 10.30 \text{ m}$

$SA_T = SA_{hs} + SA_{cyc} + SA_{cone}$
$= 157.08 + 371.99 + 240{,}33$
$= 769.40 \text{ m}^2$

Volume

$V_{hs} = \frac{2}{3}\pi r^3$
$= \frac{2}{3}\pi(5)^3$
$= 261.80 \text{ m}^3$

$$V_{cyc} = 2\pi r^2 h$$
$$= 2\pi(5)^2(7)$$
$$= 549.78 \text{ m}^3$$

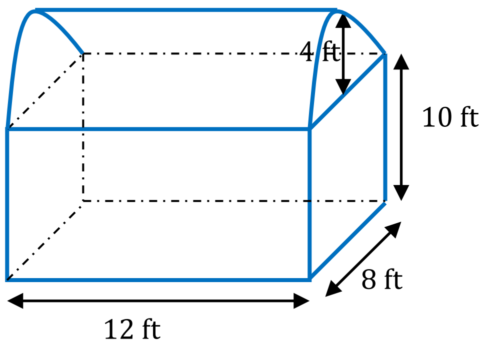
$$V_{cone} = \frac{1}{2}\pi r^2 h$$
$$= \frac{1}{2}\pi(5)^2(9)$$
$$= 353.43 \text{ m}^3$$

$$V_T = V_{hs} + V_{cyc} + V_{cone}$$
$$= 261.80 \text{ m}^3 + 549.78 \text{ m}^3 + 353.43 \text{ m}^3$$
$$= 1165.01 \text{ m}^3$$

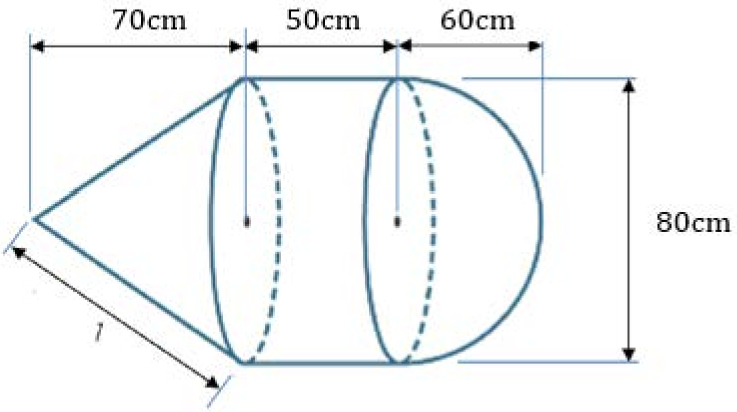
Exercise 1.7

Refer to the figure below, calculate the surface area and volume.

a.



b.



Review Answer

Answer Exercise 3.1

a. 196.8504 in b. 8 ft c. 74.5645 mi

Answer Exercise 3.2

a. 3kg b. 4.5kg c. 2.5 lbs d. 5000kg e. 1.2 t

Answer Exercise 3.3

a. 4650 in^2 b. 7.639 ft^2 c. 3600 in^2 d. 61.78 ac e. 2.02 ha

Answer Exercise 3.4

a. 8000 L b. 0.004 m^3 c. 0.009 m^3 d. 1387 in^3 e. 22.73 L

Answer Exercise 3.5

a. 20m b. 26 ft c. 30 cm d. 70 m e. 18 in

Answer Exercise 3.6

a. SA=248 m^2, V=240 m^3
b. SA=3619 in^2, V= 16286 in^3
c. SA= 804 m^2, V=2145m^3
d. SA=99 m^2, V= 58 m^3
e. SA = 72 in^2, 32 in^3

Answer Exercise 3.7

a. SA_T = 697 ft^2, V_T = 1061 ft^3
b. SA_T = 4783 cm^2, V_T = 502654 cm^3

CHAPTER 2

FUNCTION & GRAPH

Written By: En. Mohd Nawi Bin Ab. Rahman@Ismail

FUNCTION AND GRAPH

LINEAR GRAPH

Linear function is those graph is a straight line. A linear function has one in dependent variable, x and one dependent variable, y and c as the interception at y-axis.

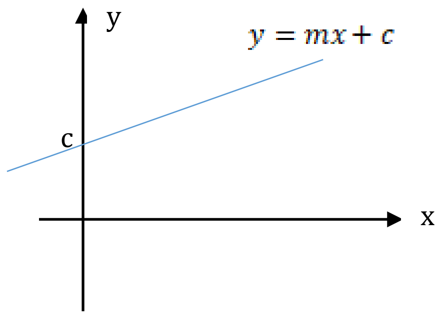
STANDARD FORM

$$y = mx + c \ldots \ldots eq\ 1$$

$$where\ ;$$

$$m = gradient\ or\ slope$$

$$c = interception\ at\ y - axis$$



GRADIENT /SLOPE, m

Gradient or slope, m at the coordinate A (x_1, y_1) and B (x_2, y_2), can be determine:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \ldots \ldots \ldots eq\ 1a$$

MIDPOINT

Midpoint between coordinate A (x_1, y_1) and B (x_2, y_2), can be determine:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \ldots . eq1b$$

QUADRATIC GRAPH

Can be determine by the power of equation is t wo.

The graph of a quadratic function is a curve called a parabola. Parabolas may open upward or downward and vary in "width" or "steepness", but they all have the same basic "U" shape

STANDARD FORM

$$y = ax^2 + bx + c \ldots \ldots eq\ 2$$

$where;$

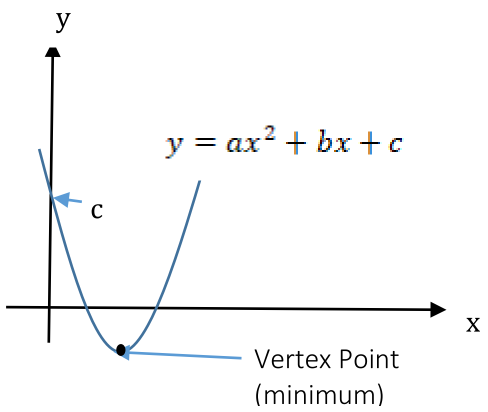
$a, b, c = real\ number$

$x = variable$

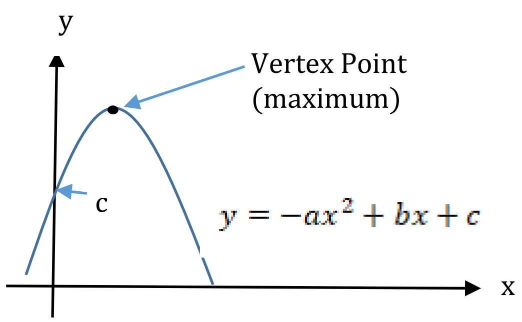
$a \neq 0$

TWO TYPE OF GRAPH

a. Open Upward (minimum point)



b. Open Downward (maximum point)



DISTANCE TWO POINT	VERTEX POINT
Distance between two point, coordinate A (x_1, y_1) and B (x_2, y_2), can be determine: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.....eq1c	Vertex point is defines the maximum and minimum point of the graph. It is can determine: $Vertex\ Point\ ; (h, k)$ $where$ $h = \frac{-b}{2a}\ ;\quad k = f(h)$

EXAMPLE QUESTION

EXAMPLE 1

A straight line passing through the coordinate P (1, -2) and Q (-3, 5). Determine:

i. The gradient of straight line.

ii. The equation of the straight line.

iii. The midpoint between coordinate P and Q

iv. The distance between coordinate P and Q

Solution

i. Gradient ; $m = \frac{5-(-2)}{-3-1}\ ;\ = \frac{7}{-4}$

Use the equation 1a : $m = \frac{y_2 - y_1}{x_2 - x_1}$

ii. General Equation straight line

$$y = mx + c\ ;\quad and\ m = \frac{7}{-4}$$

$$and\ equation\ become\ \ y = -\frac{7}{4}x + c$$

$$find\ the\ value\ of\ c\ :$$

$$Step\ 1\ \ y = -\frac{7}{4}x + c\ ,$$

Select either coordinate P or Q and replace in the Step 1

$$Step\ 2:\ -2 = -\frac{7}{4}\ (1) + c$$

$$c\ = -2 + \frac{7}{4}$$

$$= -\frac{1}{4}$$

The general equation $\boldsymbol{y = -\frac{7}{4}x - \frac{1}{4}}$

iii. Midpoint ;

$$= \left(\frac{1+(-3)}{2}, \frac{-2+5}{2}\right)$$

Use the equation 1b : $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{-2}{2}, \frac{3}{2}\right)$$

$$= \left(-1, \frac{3}{2}\right)$$

iv. Distance between P and Q

$$= \sqrt{(1-(-3)^2 + (-2-5)^2}$$

Use the equation 1c : $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$$= \sqrt{4^2 + -7^2}$$

$$= \sqrt{68}$$

$$= 8.25 unit$$

EXAMPLE 2

Find *k* if the point (2, *k*) lies on the line with slope, m = 3 and goes through the point (1,6).

Solution

From this question, given m =3

By using the equation 1a;

Use the equation 1a : $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{6-k}{1-2}$$

$$3 = \frac{6-k}{-1}$$

$$-3 = 6 - k$$

$$\boldsymbol{k = 9}$$

EXAMPLE 3

Find the value of *y* if the distance between (*y*, -2) and (-2,-14) is 13units.

Solution

From question, given value of distance = 13 unit

By using the equation 1c;

$$13 = \sqrt{(y-(-2))^2 + (-2+14)^2}$$

$$13 = \sqrt{(y+2)^2 + (12)^2}$$

$$13^2 = (y+2)^2 + 144$$

$$169 - 144 = (y+2)^2$$

$$25 = (y+2)^2$$

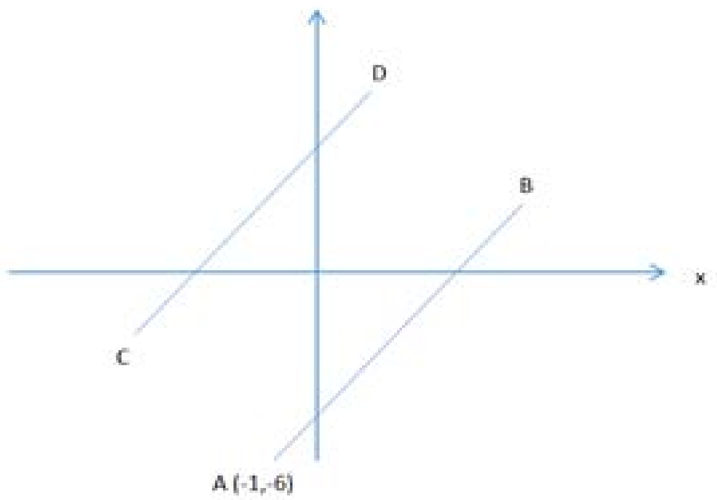
$$\sqrt{25} = y + 2 \ \ 5$$

$$= y + 2$$

$$\boldsymbol{y = 3}$$

EXAMPLE 4

Figure below two parallel line AB and CD. The equation of the straight line CD is $2y = 2x + 12$. Calculate



i. y-intercept of the straight line CD

ii. The equation of straight line AB

Solution

i. y-intercept occur at x=0

$$2y = 2x + 12 \ \ , x = 0$$

$$2y = 12$$

$$\boldsymbol{y = 6}$$

ii. AB parallel CD , from the equation $2y = 2x + 12,\ m = 1$

$$general\ equation\ y = mx + c$$

$$y = x + c\ throught\ A\ (-1, -6), to\ find\ value\ of\ c:$$

$$-6 = -1 + c$$

$$c = -5$$

$$\boldsymbol{so : Equation\ Line\ AB, y = c - 5}$$

EXAMPLE 5

Construct and solve the equation for $y = -x + 5$ and $y = \frac{1}{2}x + 2$ by using a graph.

Solution

Method 1: Using the table of value

x	-4	-3	-2	-1	0	1	2	3	4	5
$y = -x + 5$	9	8	7	6	5	4	3	2	1	0
$y = \frac{1}{2}x + 2$	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5

Method 2 : Using the concept $i. x = 0\ what\ the\ value\ of\ y\ and$

$ii. y = 0\ what\ the\ value\ of\ x$

For $y = -x + 5$;

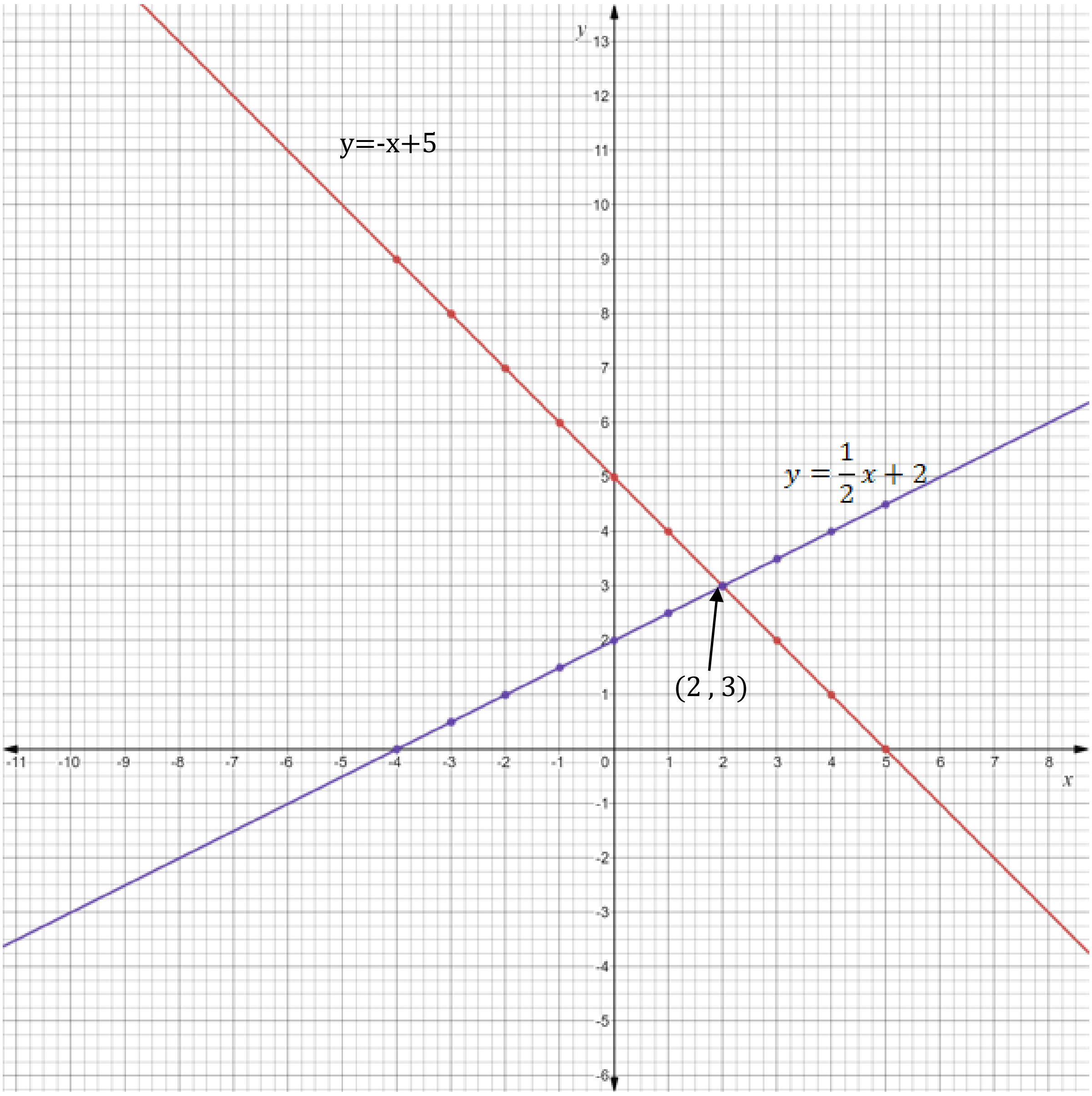
$\boldsymbol{x = 0 \quad \rightarrow\rightarrow\rightarrow y = 5}$

$\boldsymbol{y = 0 \quad \rightarrow\rightarrow\rightarrow x = 5}$

For $y = \frac{1}{2}x + 2$;

$\boldsymbol{x = 0 \quad \rightarrow\rightarrow\rightarrow y = 2}$

$\boldsymbol{y = 0 \quad \rightarrow\rightarrow\rightarrow x = -4}$


y=-x+5
$y = \frac{1}{2}x + 2$
(2 , 3)
x
y

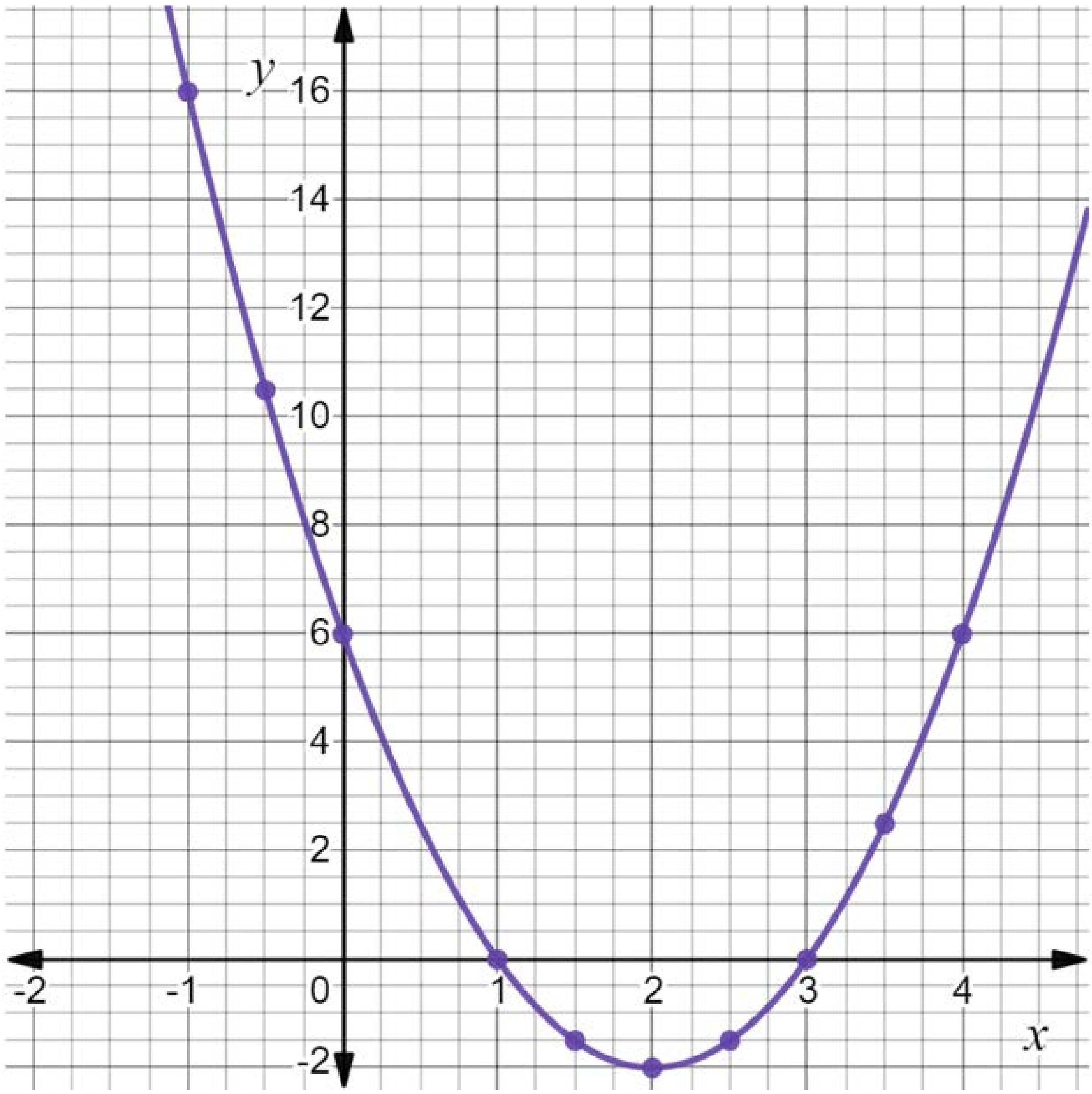
EXAMPLE 6

Construct a table of value and draw the graph for equation $y = 2x^2 - 8x + 6$ to x values from -0.5 to 4.

Solution

Table of value

x	-1	-0.5	0	1	1.5	2	2.5	3	3.5	4
$y = 2y^2 - 8x + 6$	16	10.5	6	0	-1.5	-2	-1.5	0	2.5	6



EXERCISE QUESTION

1. Given the point P (-2,5) and Q (7,1) is a straight line

   i. Find the gradient the straight line PQ

   ii. Calculate the midpoint between point P and Q

[Answer : i. m=$-\frac{4}{9}$, ii. ($\frac{5}{2}$, 3)]

2. Construct a table of value and sketch the graph for the equation $y = 3x - 12$, $-2 \leq x \leq 2$.

3. Given B $\left(-\frac{1}{2}, \frac{3}{2}\right)$ is the midpoint of a straight line that joins point A (x,-6) and C (-4, x+y). Calculate the value of x and y.

[Answer : x=3 , y=6]

4. Two graphs were intersected in two coordinate P (-2,10) and Q (1.5, -4). One of the graph was a linear graph with it gradient -4. On the other hand , another graph represents by the function of $y = 2x^2 - 3x - 4$.

   i. Calculate the distance between P and Q

   ii. Find the y-intercept for it linear graph.

   iii. Sketch both graphs.

[Answer : i.14.43unit , ii. c=2]

5. The temperature in degrees Celsius and the corresponding values in degree Fahrenheit are shown in table below.

°c	10	20	40	60	80	100
$°F$	50	68	104	140	176	212

   i. By using a suitable scale, plot the graph

   ii. Write the equation that represents relationship between these two temperature units based on the graph.

   iii. Form the graph, find the temperature in Degree at $167°F$ and at $230°F$.

6. Find the possible value of y in the following problems

   a. Given that the distance between point A (-2,3) and B (3, y-1) is 13units.

   b. Given that the distance between point G (4, y+2) and H (7, -4) is 5 units.

[Answer : a = 8 , -16, b=2,-6]

7. Given P and Q are point with coordinate (9,6) and (3, -4) respectively. R is a point on the line PQ such that PR=RQ, find the distance between P and R.

[Answer : 5.8unit]

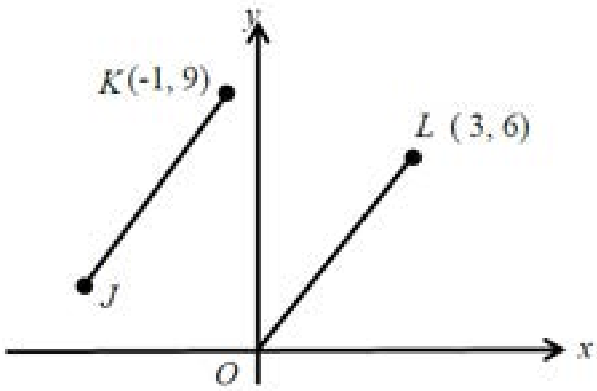
8. Given the function of graph is $y = x^2 + 2x - 8$. Determine

   i. Vertex point, x-intercept and y-intercept

   ii. Based on your answer in (i), draw a suitable graph.

[Answer : i. (-1,-9), x=2,-4,y=-8)]

9. Diagram below shows a straight line JK and OL drawn on a Cartesian plane. JK is parallel to OL.



Find :

   i. The equation of straight line JK.

   ii. The x-intercept of the straight line JK

[Answer : i. $y = 2x + 11$, x=-$\frac{11}{2}$]

10. Complete Table below for the equation $= -\frac{16}{x}$.

x	-4	-2.5	-2	-1.6	-1	1	2	2.5	4
y		6.4	8	10		-16	-8	-6.4	-4

By using a scale of 2cm to 1 unit on the x-axis and 2cm to 5 unit on the y-axis, draw the graph of $y = -\frac{16}{x}$ for $-4 \leq x \leq 4$.

i. From graph, find the value of y when x=1.2 and x when y=5

ii. Draw a suitable straight line on your graph to find all the value of x which satisfy the equation $\frac{16}{x} = 2x - 2$ for $-4 \leq x \leq 4$. State the value of x.

[Answer : y =4, 16 , i. 14.2 ≤y $\leq \mathbf{14.8}, -\mathbf{3} \leq \boldsymbol{x} \leq -\mathbf{3.5},$

$\boldsymbol{ii.}\ \boldsymbol{y} = -\mathbf{2}\boldsymbol{x} + \mathbf{2}, \boldsymbol{x} = -\mathbf{2.45}, \mathbf{3.4}$ *]*

11. Construct and solve the equation for $y = -x + 5$ and $y = \frac{1}{2}x + 2$ by using a graph.

12. Construct and solve the equation for $y = x^2 - 4 - 2$ and $y = \ x - 2$ by using a graph

CHAPTER 3

STATISTIC

Written By: Cik Nurul Amalina Binti Ibrahim

STATISTIC

3.0 Demonstrate type of data presentation

3.1.1 Presentation of data using:

i. Pictograph

Pictograph is a representation of data using images or symbols which has a title and is labelled. Pictographs represent the frequency of data while using symbols or images that are relevant to the data.

Steps to make a Pictograph:

Step 1: Collect data and then make a list or table of the data.

Step 2: Pick a symbol or picture that accurately represents the data.

Step 3: Sometimes the frequency of the data is too high. So that one symbol cannot represent one frequency.

Step 4: Create two columns that represents the category and the data. Then draw the actual symbols that represent the frequencies.

<u>**EXAMPLE 3.11**</u>

A bookseller sold 60 books on Monday, on Tuesday 50 books, on Wednesday 80 books, on Thursday 40 books, on Friday 40 books and on Saturday 70 books. Draw a pictograph for the books sold during the week.

<u>**Solution:**</u>

Days	Books sold
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

One  represents 10 books

ii. Line graph

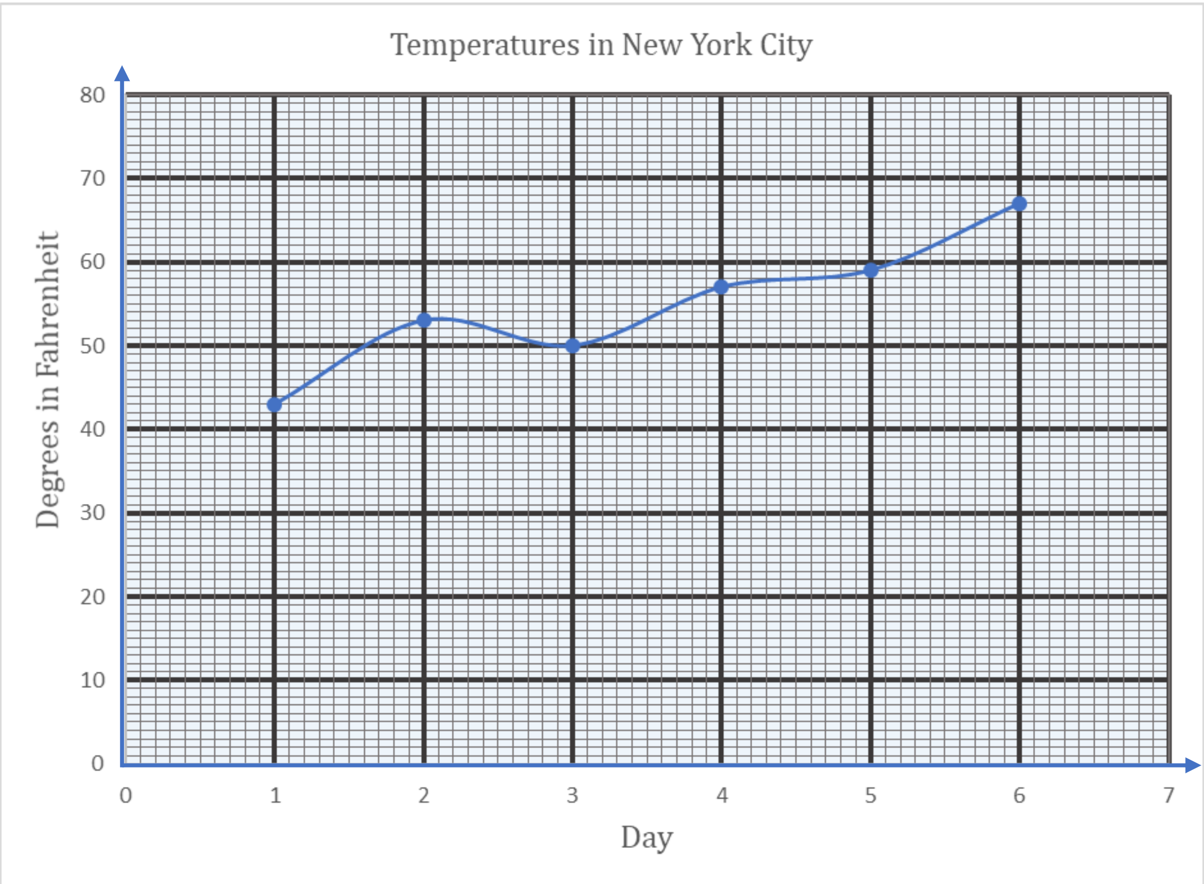
Line graph is a graph that uses lines to connect individual data points. A line graph displays quantitative values over a specified time interval. Line graphs compare two variables, one is plotted along the x-axis (horizontal) and the other along the y-axis (vertical).

EXAMPLE 3.12

The table below shows daily temperatures, in degrees Fahrenheit, for New York City recorded for 6 days. Represent the data in the form of line graph.

Day	Temperature, F
1	43
2	53
3	50
4	57
5	59
6	67

Solution:



iii. Bar Chart

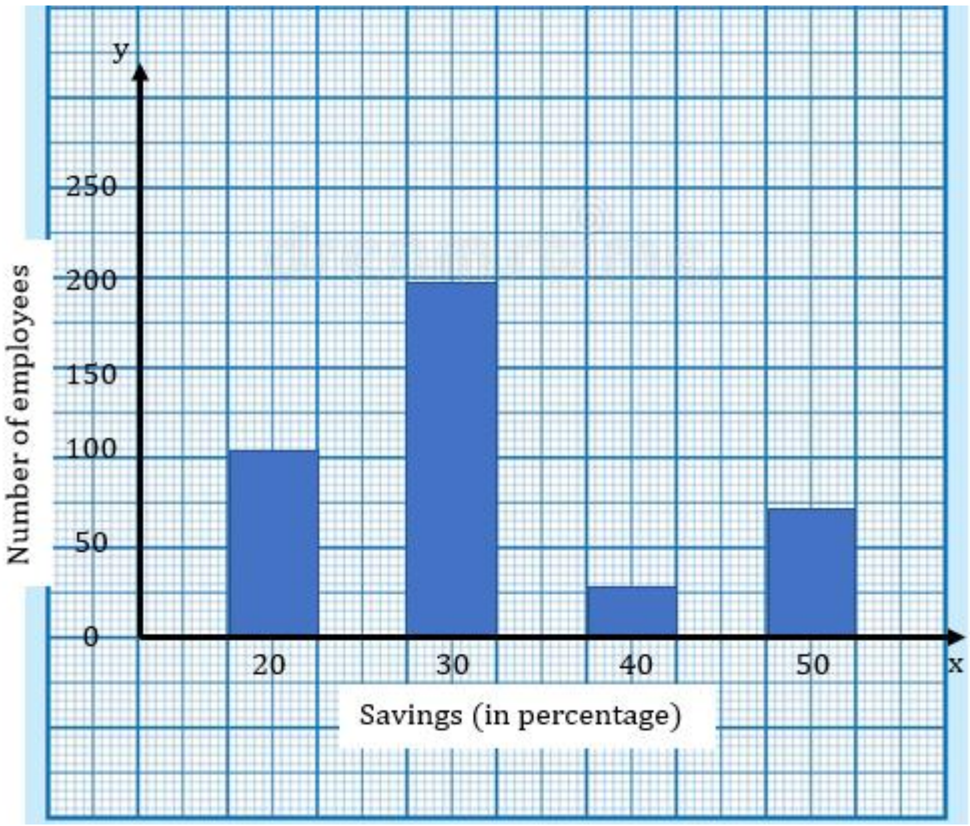
A bar chart is a type of chart that is used to represent or summarize data using bars or rectangles of equal width but different heights or lengths. They can be either vertical or horizontal graph.

Example 3.13

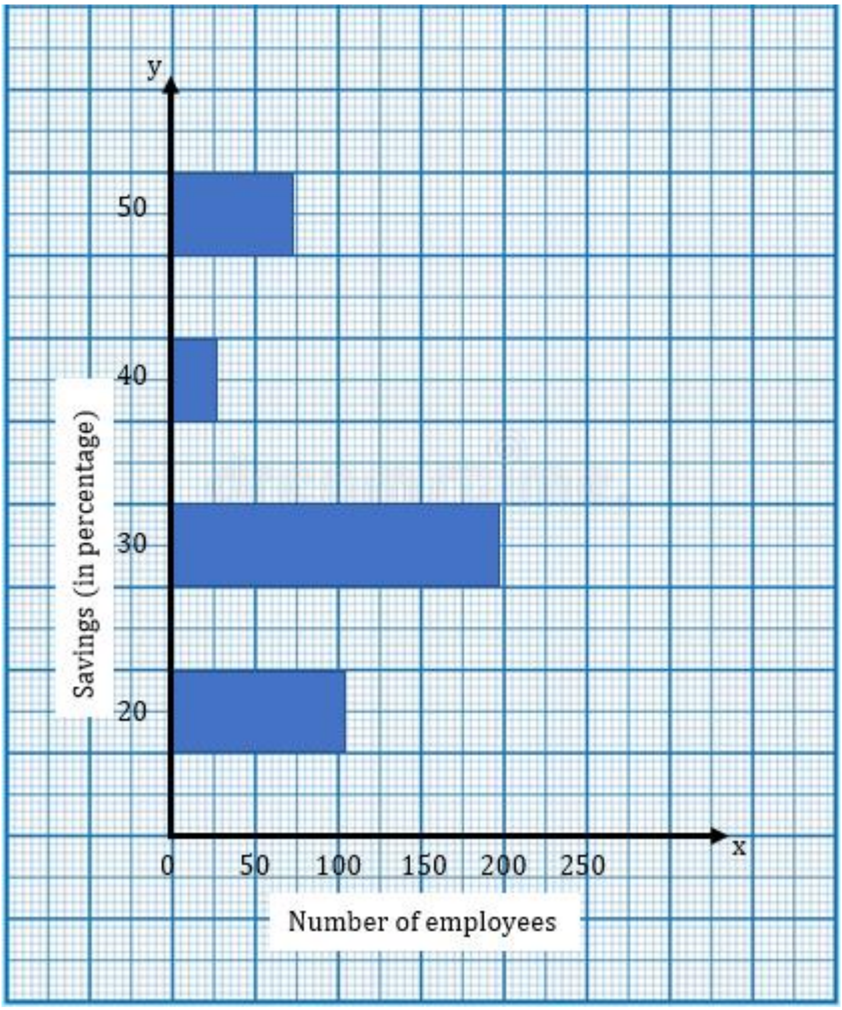
In a firm of 400 employees, the percentage of monthly salary saved by each employee is given in the following table. Represent it through a bar graph.

Savings (in percentage)	20	30	40	50
Number of employees	105	199	29	73

Solution:



Vertical bar chart/ bar graph



Horizontal bar chart/ bar graph

iv. Pie Chart

A Pie Chart is a type of graph that records data in a circular manner that is further divided into sectors for representing the data of that particular part out of the whole part. Each of these sectors or slices represent the proportionate part of the whole. Pie charts also commonly known as pie diagrams help in interpreting and representing the data more clearly.

We know that the total value of the pie is always 100%. It is also known that a circle subtends an angle of 360 . Hence, the total of all data is equal to 360 . Based on these, there are two main formulas used in pie charts:

i. To calculate the percentage of the given data: (Frequency   Total Frequency) x 100

ii. To convert the data into degrees: (Given Data   Total value of data) x 360

Example 3.14

Construct a pie chart to visually display the favorite fruits of the students in a class based on the given data.

Fruits	Apples	Bananas	Guavas	Pineapple
Number of fruits	50	40	60	60

Solution:

Step 1: Find the percentage of each value:

Mango = (45/150) x 100 = 30%

Orange = (30/150) x 100 = 20%

Plum = (15/150) x 100 = 10%

Pineapple = (30/150) x 100 = 20%

Melon = (30/150) x 100 = 20%

Step 2: Mango = (45/150) x 360 = 108

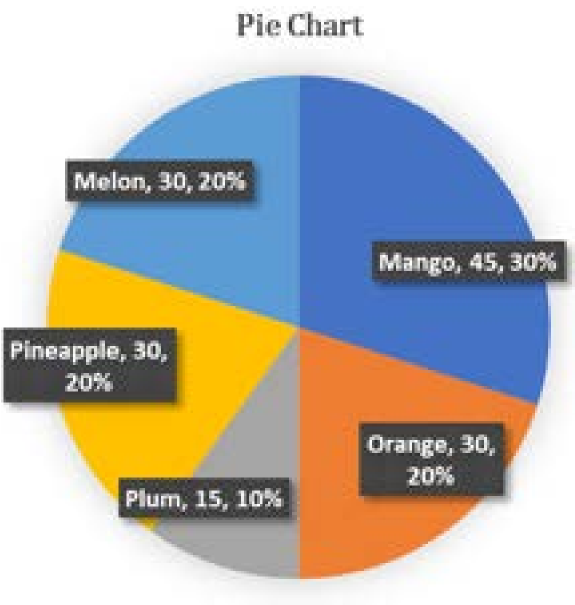
Orange = (30/150) x 360 = 72

Plum = (15/150) x 360 = 36

Pineapple = (30/150) x 360 = 72

Melon = (30/150) x 360 = 72

Step 3: With all the above degrees, with the help of a protractor draw a pie chart.



Exercise 3.1

1. A fruit seller sold the following number of different fruits as given below. Form a pictograph with the help of the given data.

Fruits	Apples	Bananas	Guavas	Pineapple
Number of fruits	50	40	60	60

2. Sarah bought a new car in 2001 for $24,000. The dollar value of her car changed each year as shown in the table below.

Year	Value, $
2001	24,000
2002	22,500
2003	19,700
2004	17,500
2005	14,500
2006	10,00
2007	5,800

Represents the data above in the form of line graph.

3. Represent the data in the table below that shows Sam's weight in kilograms for 5 months in the form of line graph.

Month	January	February	March	April	May
Weight in kg	49	54	61	69	73

4. Construct a pie chart to visually display the foods that Ana spent at the funfair.

Foods	Ice cream	Toffees	Popcorn	Candy
Number of foods	4	4	2	10

5. The number of bed-sheets manufactured by a factory during five consecutive weeks is given below.

Week	First	Second	Third	Forth	Fifth
Number of bed-sheets	130	120	135	130	150

Represent the data above in the form of bar graph.

3.2 Construct frequency table and several forms of data presentation

3.2.1 Types of statistical data

Raw data can be represented as ungrouped data and grouped data. **Ungrouped data** are listed as a sequence or in the form of a frequency table with single valued data classes but without the use of intervals. **Grouped data** refers to data which are categorized into mutually exclusive intervals.

3.2.2 Ungrouped Data

The frequency of a particular data value is the number of times the data value occurs. A frequency table is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies.

Example 3.21

Given below are marks obtained by 20 students in Mathematics Test.

21	23	19	17	12	15	15	17	17	19
23	23	21	23	25	21	19	19	19	19

Present this data in a frequency table.

Solution:

Step 1: Construct a table with two columns. The first column put the data in an ascending order.

Marks	Frequency
12	
15	
17	
19	
21	
23	
25	

Step 2: Create the second column with the frequency of each data occurrence.

Marks	Frequency
12	1
15	2
17	3
19	5
21	3
23	4
25	2

3.2.3 Grouped Data

When a data set contains many different and non-repetitive values, the data can be grouped into class intervals before the frequency distribution is constructed. The frequency table is also known as grouped frequency distribution. Data that are summarized in the form of a frequency table is known as grouped data.

i. Frequency distribution

There are **SIX** steps to follow:

Step 1: Find the largest and smallest values.

Step 2: Determine the range for the set of data.

$$Range = Highest\ Value - Lowest\ Value$$

Step 3: Choose a suitable number of classes.

Number of class,

$$k = 1 + 3.3 \log n$$

(n is the number of data)

Step 4: Determine the class size.

$$Class\ Size = \frac{Range}{Number\ of\ classes}$$

Step 5: Compute the class interval for every row.

i. Find upper class interval

$$\text{Lowest value} + (\text{Class Size} - 1)$$

ii. Find next lower class interval

Lowest value + Class Size

Step 6: State the class limit for each class and determine the frequency of the data by tabulating the frequency table.

> ***NOTE:** The value should be **rounding up!**
> Normally 3.2 would round to be 3, but in rounding up, it becomes 4

Example 3.22

The data below show the mass (kg) of 40 students in a class. Construct a frequency table for this set of data values.

55	70	57	73	55	59	64	72
60	48	58	54	69	51	63	78
75	64	65	57	71	78	76	62
49	66	62	76	61	63	63	76
52	76	71	61	53	56	67	71

Solution:

Step 1: Highest value = 78 , Lowest value = 48

Step 2: $\text{Range} = 78 - 48$

$= 30$

Step 3: Number of class,

$$k = 1 + 3.3\log(40)$$

$$=7$$

Step 4: Class size $\dfrac{30}{7}$

5

Step 5:

Lower Class Interval	Upper Class Interval
48	$48 + (7 - 1) = 54$
$48 + 7 = 55$	$55 + (7 - 1) = 61$
$55 + 7 = 62$	$62 + (7 - 1) = 68$
$62 + 7 = 69$	$69 + (7 - 1) = 75$
$69 + 7 = 76$	$76 + (7 - 1) = 82$
$76 + 7 = 83$	$83 + (7 - 1) = 89$
$83 + 7 = 90$	$90 + (7 - 1) = 96$

Step 6: Tabulate the Frequency Table.

Mass (kg)	Frequency
48 – 54	6
55 – 61	10
62 – 68	10
69 – 75	8
76 – 82	6

3.2.4 Construct Histogram, Frequency Polygon and Ogive

i. Histogram

Histogram is a two-dimensional graphical representation of a continuous frequency distribution. In histogram, the bars are placed continuously side by side with no gap between adjacent bars. The following are steps to construct a histogram.

Step 1: Draw and label x and y axis. X-axis is horizontal axis and y-axis is vertical axis.

Step 2: Represents class boundaries on the x-axis and frequency on the y-axis.

Step 3: Draw a bar extending from the lower value of each interval to the lower value of the next interval.

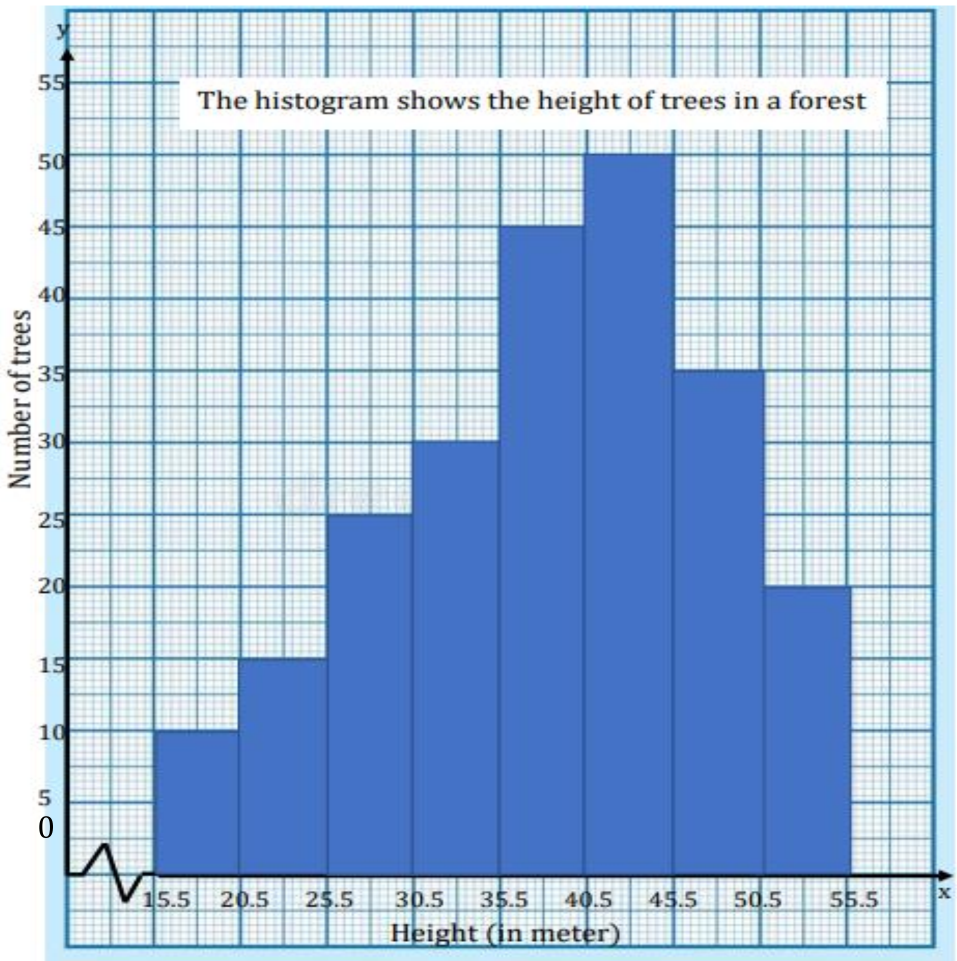
Example 3.23

The heights of trees in a forest are given as follows. Draw a histogram to represent the data.

Height (m)	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55
Number of trees	10	15	25	30	45	50	35	20

Solution:

Height (m)	Class Boundaries	Frequency, f
16 – 20	15.5 – 20.5	10
21 – 25	20.5 – 25.5	15
26 – 30	25.5 – 30.5	25
31 – 35	30.5 – 35.5	30
36 – 40	35.5 – 40.5	45
41 – 45	40.5 – 45.5	50
46 – 50	45.5 – 50.5	35
51 – 55	50.5 – 55.5	20



ii. Frequency Polygon

A frequency polygon is a line graph of class frequency plotted against class midpoint. It can be obtained by joining the midpoints of the tops of the rectangles in the histogram or can be drawn without it as well.

Step 1: Find the midpoint of each class.

$$Midpoint = \frac{Upper\ boundary + Lower\ boundary}{2}$$

Step 2: Draw x and y axis. Represents midpoint on the x-axis and frequency on the

y-axis.

Step 3: Plotting frequencies against the midpoints and joining the adjacent points with line segments.

NOTE: For a complete frequency polygon, a class with zero frequency is added before the first class and similarly a class with zero frequency is added after the last class.

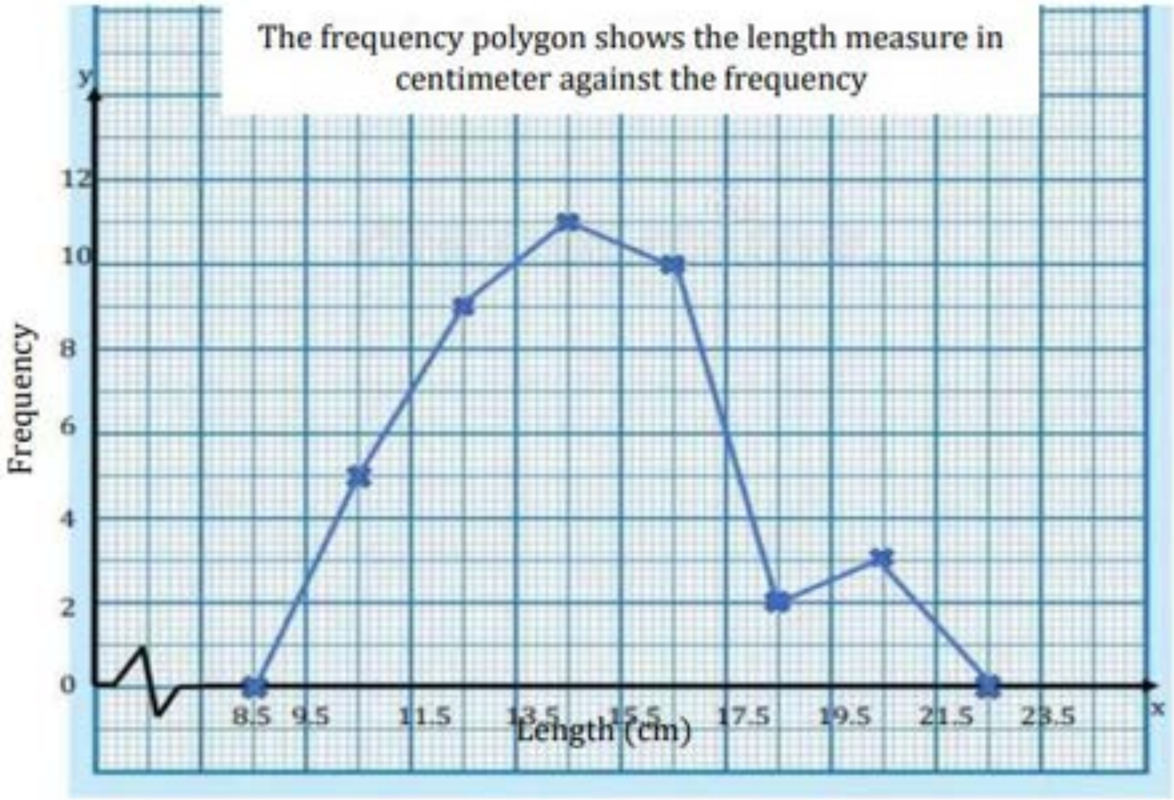
<u>**Example 3.24**</u>

Use data in the table below, then construct the frequency polygon.

Length (cm)	10 – 11	12 – 13	14 – 15	16 – 17	18 – 19	20 – 21
Frequency	5	9	11	10	2	3

Solution:

Length (cm)	Class Boundaries	Midpoint	Frequency, f
		8.5	0
10 – 11	9.5 – 11.5	10.5	5
12 – 13	11.5 – 13.5	12.5	9
14 – 15	13.5 – 15.5	14.5	11
16 – 17	15.5 – 17.5	16.5	10
18 – 19	17.5 – 19.5	18.5	2
20 – 21	19.5 – 21.5	20.5	3
		22.5	0



iii. Ogive

An ogive is also known as cumulative frequency curve. The cumulative frequency is the sum of the frequencies accumulated up to the upper boundary of a class in the distribution.

To construct an ogive, it is necessary first to form the frequency distribution table and frequency cumulative table. Represent the upper-class boundaries on x-axis and cumulative frequency on the y-axis. These points are connecting by drawing a smooth curve with free hand.

Two methods can be used to construct ogive:

i. "Less-than" method

ii. "More-than" method

Example 3.25

The table below shows the scores obtained by 60 candidates in Biological Science test.

Marks	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80
Frequency	3	8	12	14	10	6	5	2

Use data above to draw a "less-than" and "more-than" cumulative frequency curve.

Solution:

Marks	Frequency	Upper Boundary	Cumulative Frequency "less-than"	Cumulative Frequency "more-than"
		0.5	0	60
1 – 10	3	10.5	3	57
11 – 20	8	20.5	11	49
21 – 30	12	30.5	23	37
31 – 40	14	40.5	37	23
41 – 50	10	50.5	47	13
51 – 60	6	60.5	53	7
61 – 70	5	70.5	58	2
71 – 80	2	80.5	60	0

Exercise 3.2

1. Complete the table below.

i.

Class Interval	Lower Limit	Upper Limit	Lower Boundary	Upper Boundary
1 – 10	1	10	0.5	10.5
11 – 20				
21 – 30				
31 – 40				
41 – 50				

ii.

Class Interval	Lower Limit	Upper Limit	Lower Boundary	Upper Boundary
100 – 104	100			
105 – 109				
110 – 114		114		
115 – 119				
120 – 124			119.5	
125 – 129				
130 – 134				134.5

iii.

Class Interval	Lower Limit	Upper Limit	Lower Boundary	Upper Boundary
2.1 – 2.3				2.35
2.4 – 2.6				
2.7 – 2.9			2.65	
3.0 – 3.2		3.2		
3.3 – 3.5	3.3			

2. These data represent the record high temperatures in F for each of the 50 states. Construct a grouped frequency distribution for the data.

112	100	127	120	134	118	105	110	109	112
110	118	117	116	118	122	114	114	105	109
107	112	114	115	118	117	118	122	106	110
116	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

3. Table below shows the distribution of marks of 40 students in Mathematics Test. From the table below, draw a histogram and frequency polygon.

Marks	Number of students
30 – 39	6
40 - 49	8
50 - 59	12
60 – 69	8
70 – 79	5
80 - 89	1

4. The following distribution gives the daily income of 50 workers of a factory.

Daily Income (RM)	Number of workers
100 – 120	12
120 – 140	14
140 – 160	8
160 – 180	6
180 – 200	10

Convert the distribution above to a less-than type cumulative distribution and draw its ogive.

5.

Production yield	Number of farms
50 – 55	2
55 – 60	8
60 – 65	12
65 – 70	24
70 – 75	38
75 – 80	16

Table above shows the production yield per hectare of wheat of 100 farms of a village. Construct a table of more-than type distribution and draw its ogive.

6.

125	142	146	158	162	171	129	143	147	160
162	173	131	148	151	159	164	140	148	151
149	136	150	161	152	158	165	155	159	160

The data above shows the height, in cm, of 30 students.

i. Construct a frequency table for the data above by choosing a suitable class interval such that there will be 7 class intervals. Start your class interval with the lowest data.

ii. Using a correct scale, draw a histogram and frequency polygon.

3.3 Central Tendency and Dispersion

3.3.1 Calculate Mean, Median and Mode for Ungrouped Data

i. Mean

The mean of a set of an ungrouped data can be obtained by adding all the values of the data and dividing the sum by the total number of values of the data.

Mean, $\bar{x}$ $\frac{x}{n}$

where, n = total number of values of the data

x = sum of values of x

$\bar{x}$ = mean of data

❖ If a set of ungrouped data is given in a frequency distribution table, the formula to find the mean is as follow.

$$\text{Mean, } \bar{x} \quad \frac{fx}{f}$$

where, fx = sum of all values of the data

f = total values of frequency

<u>**Example 3.31**</u>

Calculate the mean of each of the following sets of data.

i. 5 , 6 , 2 , 4 , 7 , 8 , 3 , 5 , 6 , 6

Solution:

$$\bar{x} \quad \frac{x}{n}$$

$$\frac{5 \quad 6 \quad 2 \quad 4 \quad 7 \quad 8 \quad 3 \quad 5 \quad 6 \quad 6}{10}$$

$$\frac{52}{10}$$

$$5.2$$

ii.

No. of Class	0	1	2	3	4	5	6	7	8	9
Frequency	3	4	6	7	10	6	5	5	3	1

Solution:

No. of Class, x_i	Frequency, f_i	$f_i x_i$
0	3	0
1	4	4
2	6	12
3	7	21
4	10	40
5	6	30
6	5	30
7	5	35
8	3	24
9	1	9
	f 50	$f_i x_i$ 205

Mean,

$$\bar{x} \quad \frac{fx}{f}$$

$$\frac{205}{50}$$

$$4.1$$

ii. Mode

The mode of a set of ungrouped data can be determined by identifying the value which occurs most frequently.

<u>Example 3.32</u>

Calculate the mode for the following data.

i. 70 , 76 , 80 , 85 , 85 , 85 , 95 , 96 , 100 , 108

Solution:

Mode = 85 (From the observation, 85 occurs maximum number of times)

ii.

Size of the winter coat	38	39	40	42	43	44	45
Total	33	11	22	55	44	11	22

Solution:

Solution:

Mode = Mode for the size of winter coat is 42. (Size 42 has the greatest frequency)

iii. Median

The median of a set of ungrouped data is the value in the middle position of the set when the values of the data are arranged in ascending order.

If the total number of values of the data is even, there will be two values in the middle. Thus, the median is the mean of the two middle values.

If n is odd, then the middle position $\frac{n \quad 1}{2}$.

If n is even, then the two middle positions are at positions $\frac{n}{2}$ and $\frac{n}{2} \quad 1$.

Example 3.33

1. Calculate the median for the set of data below.

   i. 7, 16 , 10 , 6 , 11 , 25 , 11 , 13 , 16

   Solution:

   Arrange the numbers in ascending order.

   6 , 7 , 10 , 11 , $\boxed{11}$, 13 , 16 , 18 , 25

   Since 11 is in the middle, therefore the median is 11.

   ii. 2 , 3 , 4 , 5 , 5 , 6 , 6 , 6 , 7 , 8

   Solution:

   2 , 3 , 4 , 5 , $\boxed{5}$, $\boxed{6}$, 6 , 6 , 7 , 8

   Median, $\frac{5 \quad 6}{2}$

   5.5

2. The frequency distributions of seed yield of 50 sesamum plants are given below. Find the median for the data.

Seed yield	3	4	5	6	7
Frequency	4	6	15	15	10

Solution:

Total number of values of the data = 50 (Even)

$\frac{50}{5}$

25

Seed yield	3	4	5	6	7
Frequency	4	6	15	15	10
Cumulative Frequency	4	10	$\boxed{25}$	40	50

The middle position is at 25. Then, the value at position 25 is 5.

The median is 5.

Exercise 3.3

1. Determine the mean for the following data.

   i. 3 , 4 , 4 , 5 , 7

   ii. 17 , 18 , 16 , 17 , 17 , 14 , 22 , 15 , 16 , 17 , 14 , 12

   iii. 6.2 , 5.7 , 9.4 , 2.1 , 8.4 , 12.3 , 10.7 , 7.6

2. Find the median for the following set of data.

   i. 45 , 32 , 37 , 46 , 39 , 36 , 41 , 48 , 36

   ii. 1.32 , 1.34 , 1.25 , 1.35 , 1.32 , 1.35

   iii. 21 , 30 , 22 , 16 , 24 , 28 , 16 , 17

3. Find the mode for this set of data.

   i. 7 , 4 , 9 , 5 , 8 , 3 , 8 , 8 , 2

   ii. 23 , 24 , 24 , 26 , 27 , 28 , 30 , 24 , 29 , 24 , 27

   iii. 5.3 , 5.7 , 5.9 , 5.4 , 4.5 , 5.7 , 5.8 , 5.7

4. The marks obtained by 50 students in an examination is recorded in the following table.

Marks	No. of students
4	2
5	3
6	9
7	14
8	13
9	7
10	2

   Calculate:

   i. the mean

   ii. the mode

   iii. the median

5. Given that the mean of the set of data 5 , 7 , 9 , 12 , x , 19 , 9 , 10 is 11, find the value of x.

6. The following frequency distribution table shows the number of pens each student has for a group of students.

No. of pencils	1	2	3	4	5
No. of students	4	7	5	y	1

Given that the mean number of pens is 2.5, calculate the value of y.

3.3.2 Calculate Mean, Median and Mode for Grouped Data by Using Formula and Graph

i. Mean

When the values of a set data are grouped into classes in a frequency table, the value that is used to represent all the values of the data in a class is the midpoint of the class.

$$\text{Mean, } \bar{x} \quad \frac{fx}{f}$$

where, x = class midpoint
f = class frequency

<u>Example 3.34</u>

1. The age of children in a primary school were recorded in the table below.

Age	5 – 6	7 – 8	9 – 10
Frequency	29	40	38

Find the median for the age of children.

Solution:

Age	Midpoint, x_i	Frequency, f_i	$f_i x_i$
5 – 6	5.5	29	159.5
7 – 8	7.5	40	300
9 – 10	9.5	38	361
		f 107	$f_i x_i$ 820.5

$$\text{Mean, } \bar{x} \quad \frac{fx}{f} \quad \frac{820.5}{107}$$

$$7.67$$

2. Calculate the mean for the data in table below.

Marks	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Frequency	12	21	30	19	8

Solution:

Marks	Midpoint, x_i	Frequency, f_i	$f_i x_i$
40 – 49	44.5	12	534
50 – 59	54.5	21	1144.5
60 – 69	64.5	30	1935
70 – 79	74.5	19	1415.2
80 – 89	84.9	8	679.2
		f 90	$f_i x_i$ 5707.9

Mean, $\bar{x}$ $\frac{fx}{f}$

$\frac{5707.9}{90}$

63.42

ii. Mode

When data has been grouped and a frequency curve is drawn to fit the data, the mode is the value of x corresponding to the maximum point on the curve. The class which has the largest frequency is called the modal class.

An estimate of the mode can be obtained by using two methods:

i. By using formula

$$\text{Mode} \quad L \quad \frac{d_1}{d_1 \quad d_2} \quad c$$

where,

L = Lower class boundary of modal class

d_1 = frequency of modal class – frequency of the class before

d_2 = frequency of modal class – frequency of the class after

c = width of the modal class

ii. From a histogram

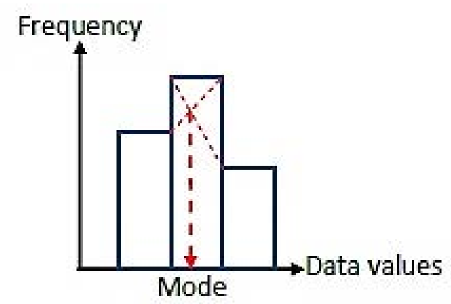
Steps to estimate the mode of data

Step 1: Identify the bar representing the modal class.

Step 2: Draw lines to join the top vertices of the bar to the top vertices of the adjacent bars on the left and right.

Step 3: Identify the point of intersection of the lines drawn in Step 2.

Step 4: Determine the value on the horizontal axis which corresponds to the point of intersection in Step 3. The value is the estimated mode.



Example 3.35

1. Find the mode for the frequency distribution table below.

Class Interval	51 – 55	56 – 60	61 – 65	66 – 70
Frequency	2	7	8	4

Solution:

Method (i) – Formula

$$\text{Mode} = L + \left(\frac{d_1}{d_1 + d_2}\right) c$$

Modal class $= 61 - 65$ (Class interval with highest frequency)

$L = 60.5$

$d_1 = 8 - 7 = 1$

$d_2 = 8 - 4 = 4$

$c = 5$

$$\text{Mode} = 60.5 + \left(\frac{1}{1 + 4}\right) 5$$

$$= 61.5$$

<u>Method (ii) – Histogram</u>

Based on histogram above, the mode value is 61.5.

2. The following table shows the mass gain (in kg) for 100 cows during a certain period. Calculate the mode mass gain of the cows.

Mass gain (kg)	5 – 9	10 – 14	15 – 19	20 – 24	25 - 29	30 – 34
Frequency	2	29	37	16	14	2

Solution:

<u>Method (i) - Formula</u>

Mode $\quad L \quad \frac{d_1}{d_1 \quad d_2} \; c$

Modal class $\quad 15 \quad 19$ (Class interval with highest frequency)

$L \quad 14.5$

$d_1 \quad 37 \quad 29 \quad 8$

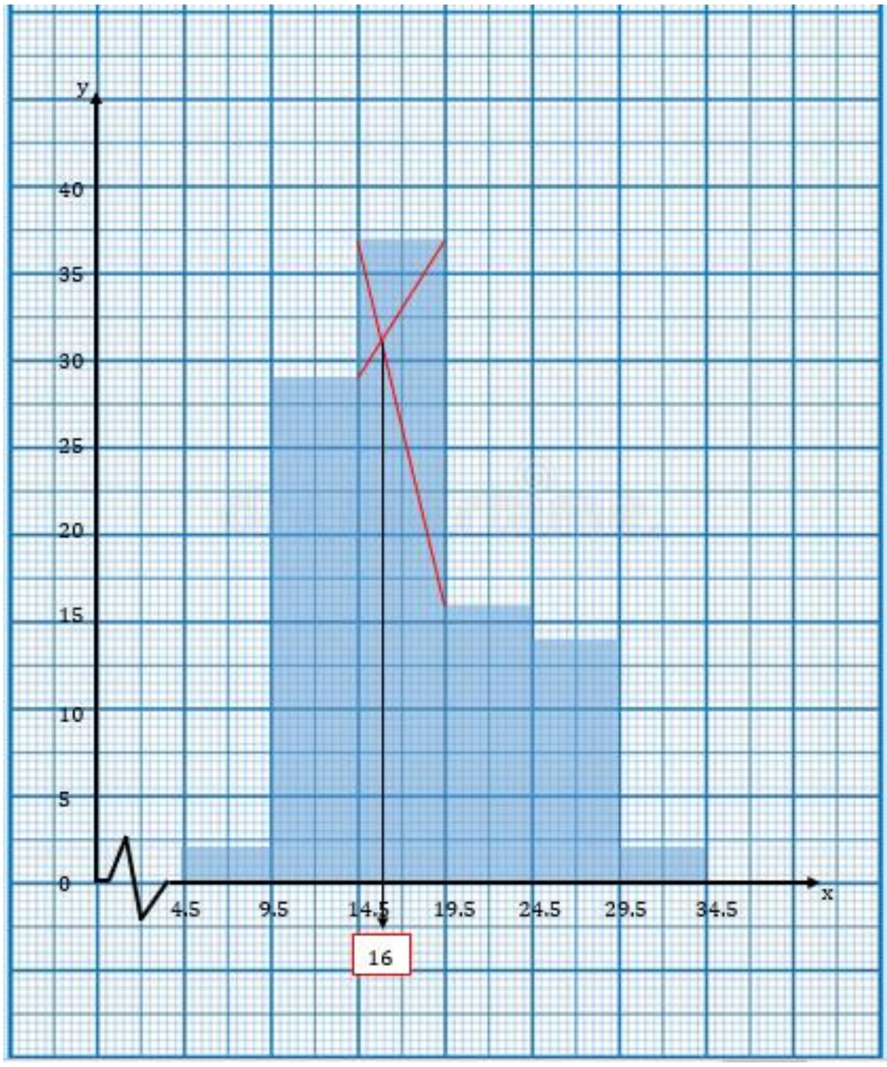
$d_2 \quad 37 \quad 16 \quad 21$

$c \quad 5$

$$\text{Mode} \quad 14.5 \quad \frac{8}{8 \quad 21} \quad 5$$

$$15.88$$

<u>**Method (ii) – Histogram**</u>



Based on histogram above, the mode value is 61.5.

iii. Median

Median is the value which occupies the middle position when all the data are arranged in ascending order. Once the information has been grouped, the originality of raw data is lost.

Hence, we can only estimate a value for the median by using:

i. By using formula

$$\text{Median, } m \quad L \quad \frac{\frac{N}{2} \quad F}{f} \quad C$$

where,

L = Lower class boundary of the median class

N = Sum of frequency

F = Cumulative frequency before median class

f = frequency of median class

C = Size of median class

ii. From a cumulative frequency curve / Ogive

The median of the set of N grouped data can be estimated from the ogive by reading the corresponding $\left(\frac{N}{2}\right)^{th}$ value of the data.

Example 3.36

1. Find the median for the distribution of examination marks given below.

Marks	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99
No. of Students	8	87	190	304	211	85	20

Solution:

Marks	No. of Students, f_i	Cumulative Frequency
30 – 39	8	8
40 – 49	87	95
50 – 59	190	285
60 – 69	304	589
70 – 79	211	800
80 – 89	85	885
90 – 99	20	905
	f 905	

Total frequency, $N \quad 905$ (Odd Number)

Middle position of the data, $\frac{905 \quad 1}{2} \quad 453$

The median class is 60 – 69.

$$\text{Median, } m \quad L \quad \frac{\frac{N}{2} \quad F}{f} \quad C$$

$$59.5 \quad \frac{\frac{905}{2} \quad 285}{304} \quad 10$$

$$59.5 \quad \frac{452.5 \quad 285}{304} \quad 10$$

$$65.01$$

2. Calculate the median for the data below.

Age Groups	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	40	53	58	64	72	49	36	24

Solution:

Age Groups	Frequency	Cumulative Frequency
0 – 10	40	40
10 – 20	53	93
20 – 30	58	151
30 – 40	64	215
40 – 50	72	287
50 – 60	49	336
60 – 70	36	372
70 – 80	25	396

Total frequency, N 396 (Even Number)

Middle position of the data, $\frac{396}{2}$ 198

The median class is 30 – 40.

$$\text{Median, } m \quad L \quad \frac{\frac{N}{2} \quad F}{f} \quad C$$

$$29.5 \quad \frac{\frac{396}{2} \quad 151}{64} \quad 11$$

$$37.58$$

3. Find the median of the data below by plotting an ogive.

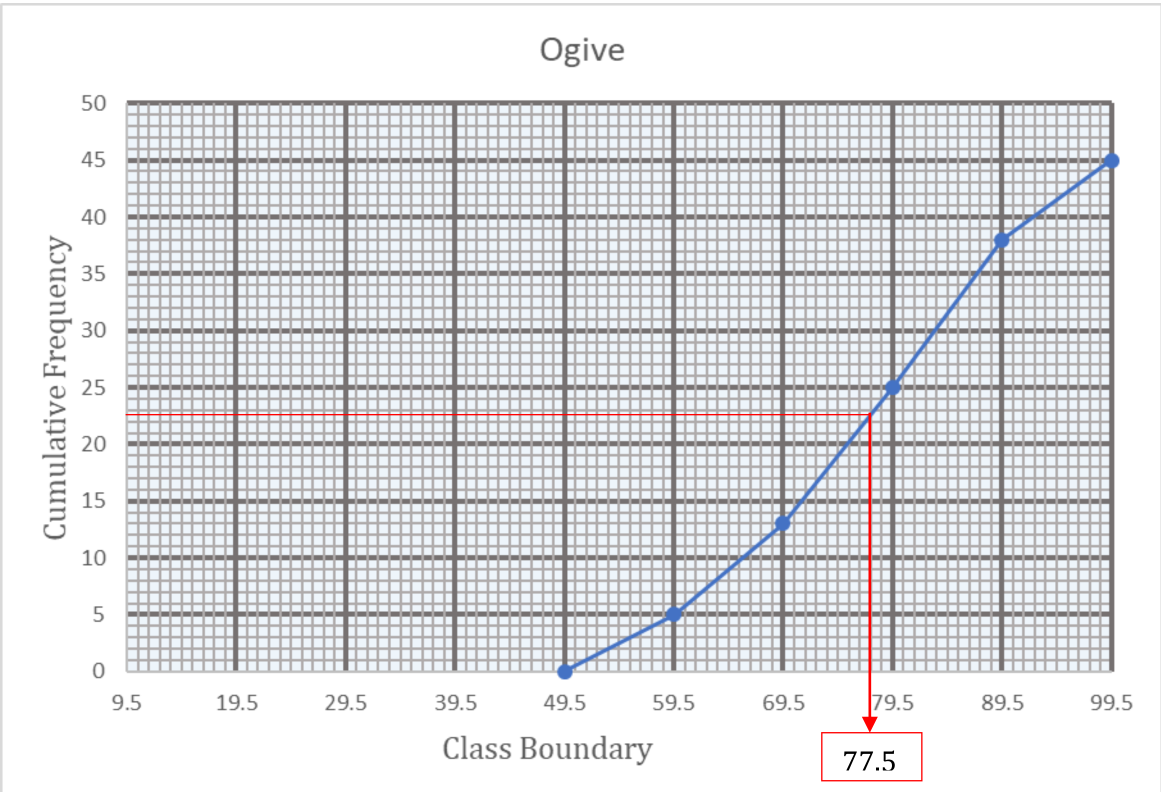
Class	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99
Frequency	5	8	12	13	7

Solution:

Upper Boundary	Cumulative Frequency
49.5	0
59.5	5
69.5	13
79.5	25
89.5	38
99.5	45

Total frequency = 45

$$\frac{N}{2} \quad \frac{45}{2} \quad 22.5$$



From the ogive, the estimated median is 77.5.

Exercise 3.4

1. The table below shows the data on the number of patients visiting a hospital in a month. Find the mean of patients visiting the hospital in a day.

Number of patients	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of days visiting hospital	2	6	9	7	4	2

2. A survey on the heights (in cm) of 50 girls of class X was conducted at a school and the following data was obtained.

Height (in cm)	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170
Number of girls	2	8	12	20	8

Find the mode and median for the data above.

3. The data below shows the ages (years) of 50 persons in Taman Kifayah.

Ages (years)	45 – 48	49 – 52	53 – 56	57 – 60	61 – 64	65 – 68	69 – 72
Frequency	6	8	12	6	9	7	2

Represent the data by drawing an ogive and then find the median from the ogive.

4. The table below shows the frequency distribution of age of 100 participants in a competition.

Age (years)	No. of participants
11 – 15	9
16 – 20	11
21 – 25	18
26 – 30	27
31 – 35	18
36 – 40	12
41 – 45	5

i. Draw a histogram to represent the data.

ii. From the histogram, find the mode.

5. The following table shows the mass distribution of watermelons transported by a lorry to the Farmer's Market. Calculate:

   i. the median mass of the watermelon

   ii. the mode mass of the watermelon

Mass (kg)	Number of watermelons
401 – 500	128
501 – 600	295
601 – 700	582
701 – 800	437
801 – 900	212
901 – 1000	162
1001 – 1100	108
1101 – 1200	76

6. The table shows the monthly wage distribution for 60 workers in factory.

Wages (RM)	Number of workers
300 – 399	5
400 – 499	11
500 – 599	12
600 – 699	18
700 – 799	8
800 – 899	4
900 - 999	2

   i. Find the median by calculation.

   ii. Draw an ogive and find the estimate median.

   iii. Based on your answer for the median value at (i) and (ii), which answer is more accurate?

3.3.3 Calculate Mean Deviation, Variance and Standard Deviation

MEASURE OF DISPERSION	
Ungrouped Data	**Grouped Data**
Mean Deviation $E = \frac{\sum_{i=1}^{n} f\lvert x - \bar{x}\rvert}{\sum f}$	**Mean Deviation** $E = \frac{\sum_{i=1}^{n} f\lvert x - \bar{x}\rvert}{\sum f}$
Variance $s^2 = \frac{\sum_{i=1}^{n} f\left(x - \bar{x}\right)^2}{\sum f}$	**Variance** $s^2 = \frac{\sum_{i=1}^{n} f\left(x - \bar{x}\right)^2}{\sum f}$
Standard Deviation $s = \sqrt{\frac{\sum_{i=1}^{n} f\left(x-\bar{x}\right)^2}{\sum f}}$ **Or** $s = \sqrt{s^2}$	**Standard Deviation** $s = \sqrt{\frac{\sum_{i=1}^{n} f\left(x-\bar{x}\right)^2}{\sum f}}$ **Or** $s = \sqrt{s^2}$

<u>Example 3.37</u>

Find the mean deviation for the following set of data.

i. 6 , 7 , 10 , 12 , 13 , 4 , 8 , 12

Solution:

$$\text{Mean, } \bar{x} = \frac{6 + 7 + 10 + 12 + 13 + 4 + 8 + 12}{8}$$

$$= \frac{72}{8}$$

$$= 9$$

Mean Deviation,

$$E = \frac{|6-9| + |7-9| + |10-9| + |12-9| + |13-9| + |4-9| + |8-9| + |12-9|}{8}$$

$$= \frac{3 + 2 + 1 + 3 + 4 + 5 + 1 + 3}{8}$$

$$\frac{22}{8}$$
$$2.75$$

ii.

Number of class	1	3	5	7	9	11	13	15
Frequency	3	3	4	14	7	4	3	4

Solution:

No. of Class, x_i	Frequency, f_i	$x_i f_i$	$\lvert x_i \quad \bar{x} \rvert$	$\lvert x_i \quad \bar{x} \rvert f_i$
1	3	3	7	21
3	3	9	5	15
5	4	20	3	12
7	14	98	1	14
9	7	63	1	7
11	4	44	3	12
13	3	39	5	15
15	4	60	7	28
	$f \quad 42$	$fx \quad 336$		$\lvert x_i \quad \bar{x} \rvert f_i \quad 124$

Mean, $\bar{x} \quad \frac{fx}{f} \quad \frac{336}{42}$, Mean deviation, $E \quad \frac{\sum_{i\ 1}^{n} f \lvert x \quad \bar{x} \rvert}{f}$

8 $\qquad\qquad$ $\frac{124}{42}$

2.95

iii.

Age Group	15 – 25	25 – 35	35 – 45	45 – 55
Number of People	25	54	34	20

Solution:

Age Group	No. of People, f_i	Midpoint x_i	$x_i f_i$	$\lvert x_i \quad \bar{x} \rvert$	$\lvert x_i \quad \bar{x} \rvert f_i$
15 – 25	25	20	500	13.68	342
25 – 35	54	30	1620	3.68	198.72
35 – 45	34	40	1360	6.32	214.88
45 – 55	20	50	1000	16.32	326.40
	$f \quad 133$		$fx \quad 4480$		$\lvert x_i \quad \bar{x} \rvert f_i \quad 1082$

$$\text{Mean, } \bar{x} = \frac{\sum fx}{\sum f} = \frac{4480}{133} = 33.68 \quad , \quad \text{Mean deviation, } E = \frac{\sum_{i=1}^{n} f\left|x - \bar{x}\right|}{\sum f} = \frac{1082}{133} = 8.14$$

Example 3.38

Find the variance and standard deviation of the following data.

i. 5 , 15 , 25 , 35 , 45 , 55

Solution:

x_i	$\left\|x_i - \bar{x}\right\|$	$(x_i - \bar{x})^2$
5	25	625
15	15	225
25	5	25
35	5	25
45	15	225
55	25	625
		$\sum (x_i - \bar{x})^2 = 1750$

$$\text{Mean, } \bar{x} = \frac{5+15+25+35+45+55}{6} = \frac{180}{6} = 30$$

$$\text{Variance, } s^2 = \frac{\sum_{i=1}^{n} f(x - \bar{x})^2}{\sum f} = \frac{1750}{6} = 291.67$$

$$\text{Standard deviation, } s = \sqrt{s^2} = \sqrt{291.67} = 17.08$$

ii.

Size	20	21	22	23	24
Frequency	6	4	5	1	4

Solution:

Size x_i	Frequency f_i	$x_i f_i$	$\left\|x_i \quad \bar{x}\right\|$	$x_i \quad \bar{x}^{\,2}$
20	6	120	1.65	2.72
21	4	84	0.65	0.42
22	5	110	0.35	0.12
23	1	23	1.35	1.82
24	4	96	2.35	5.52
	$f \quad 20$	$fx \quad 433$		$x_i \quad \bar{x}^{\,2} \quad 10.6$

Mean, $\bar{x} \quad \dfrac{fx}{f} \quad \dfrac{433}{20}$

21.65

Variance, $s^2 \quad \dfrac{{}^{n}_{i\ 1} f \ \ x \quad \bar{x}^{\,2}}{f}$

$\dfrac{10.6}{20}$

0.53

Standard deviation, $s = \sqrt{s^2}$

$\sqrt{0.53}$

0.73

iii.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	10	20	30	50	40	30

olution:

Age Group	No. of People, f_i	Midpoint x_i	$x_i f_i$	$\left\|x_i \quad \bar{x}\right\|$	$\left\|x_i \quad \bar{x}\right\| f_i$
0 – 10	10	5	50	30	300
10 – 20	20	15	300	20	400
20 – 30	30	25	750	10	300
30 – 40	50	35	1750	0	0
40 – 50	40	45	1800	10	400
50 – 60	30	55	1650	20	600
	$f \quad 180$		$fx \quad 6300$		$\left\|x_i \quad \bar{x}\right\| f_i \quad 2000$

$$\text{Mean, } \bar{x} \quad \frac{fx}{f} \quad \frac{6300}{180}$$

$$35$$

$$\text{Variance, } s^2 \quad \frac{\sum_{i\ 1}^{n} f\ \ x\ \ \bar{x}^{\ 2}}{f}$$

$$\frac{2000}{180}$$

$$11.11$$

Standard deviation, $s = \sqrt{s^2}$

$$\sqrt{11.11}$$

$$3.33$$

Exercise 3.5

1. Calculate the mean deviation, variance and standard deviation for the following data.

   i. 57 , 64 , 43 , 67 , 49 , 59 , 44 , 47 , 61 , 59

   ii. 6.3 , 4.2 , 8.4 , 3.3 , 4.0 , 2.1 , 3.8 , 4.2 , 5.3 , 5.9

   iii. 8 , 6 , 7 , 2 , 7 , 8 , 9 , 6 , 5 , 8

2. Find the variance and standard deviation of the sample data below:

Weight (kg)	60 – 62	63 – 65	66 – 68	69 – 71	72 – 74
Frequency	5	18	42	27	8

3. Calculate mean, variance and standard deviation of the following frequency distribution table.

Classes	Frequency
1 – 10	11
10 – 20	29
20 – 30	18
30 – 40	4
40 – 50	5
50 - 60	3

4. The frequency distributions of seed yield of 50 hibiscus plants are given below. Find the standard deviation.

Seed yield	2.5 – 3.5	3.5 – 4.5	4.5 – 5.5	5.5 – 6.5	6.5 – 7.5
No. of plants	4	6	15	15	10

5. The mean of a set numbers 6 , 7 , 10 , 11 , 11 , 13 , 16, 18 and 25 is 13. Calculate the variance and standard deviation for this data.

6. Find the mean deviation, variance and standard deviation for the table below.

Class	2	3	4	5	6	7
Frequency	6	10	15	8	3	10

REVIEW ANSWER

EXERCISE 3.2

1. i. Lower limit: 11, 21, 31, 41
   Upper limit: 20, 30, 40, 50
   Lower boundary: 10.5, 20.5, 30.5, 40.5
   Upper boundary: 20.5, 30.5, 40.5, 50.5

   ii. Lower limit: 105, 110, 115, 120, 125, 130
   Upper limit: 104, 109, 119, 124, 129, 134
   Lower boundary: 99.5, 104.5, 109.5, 114.5, 124.5, 129.5
   Upper boundary: 104.5, 109.5, 114.5, 119.5, 124.5, 129.5

   iii. Lower limit: 2.1, 2.4, 2.7, 3.0
   Upper limit: 2.3, 2.6, 2.9, 3.5
   Lower boundary: 2.05, 2.35, 2.95, 3.25
   Upper boundary: 2.65, 2.95, 3.25, 3.55

2.

Class limit	Frequency
100 – 104	2
105 – 109	8
110 – 114	18
115 – 119	13
120 – 124	7
125 – 129	1
130 – 134	1

3.

Marks	Number of students	Class Boundaries	Midpoints
30 – 39	6	29.5 – 39.5	34.5
40 - 49	8	39.5 – 49.5	44.5
50 - 59	12	49.5 – 59.5	54.5
60 – 69	8	59.5 – 69.5	64.5
70 – 79	5	69.5 – 79.5	74.5
80 - 89	1	79.5 – 89.5	84.5

4.

Daily Income (RM)	Number of workers	Upper Boundary	Cumulative Frequency "less-than"
		99.5	0
100 – 120	12	120.5	12
120 – 140	14	140.5	26
140 – 160	8	160.5	34
160 – 180	6	180.5	40
180 – 200	10	200.5	50

5.

Production yield	Number of farms	Upper Boundary	Cumulative Frequency "more-than"
		49.5	100
50 – 55	2	55.5	98
55 – 60	8	60.5	90
60 – 65	12	65.5	78
65 – 70	24	70.5	54
70 – 75	38	75.5	16
75 – 80	16	80.5	0

6.

Class Interval	Frequency	Class Boundary	Midpoint
125 – 131	3	124.5 – 131.5	128
132 – 138	1	131.5 – 138.5	135
139 – 145	3	138.5 – 145.5	142
146 – 152	9	145.5 – 152.5	149
153 – 159	5	152.5 – 159.5	156
160 – 166	7	159.5 – 166.5	163
167 – 173	2	166.5 – 173.5	170

<u>EXERCISE 3.3</u>

1. i. $\bar{x}$ 4.6
   ii. $\bar{x}$ 16.25
   iii. $\bar{x}$ 7.8

2. i. Median = 39
   ii. Median = 1.33
   iii. Median = 21.5

3. i. Mode = 8
   ii. Mode = 24
   iii. Mode = 5.7

4. i. $\bar{x}$ 7 , ii. Mode = 7 , iii. Median = 5.7
5. x 17
6. y = 3

EXERCISE 3.4

1. $\bar{x}$ 28.67
2. Mode = 154 , Median = 151.5
3. Median = 56.1 (from the ogive)
4. Mode = 28 (from the histogram)
5. i. Median = 699.141

   ii. Mode = 666.435
6. Median = 610.61 , method by calculation is more accurate

EXERCISE 3.5

1. i. E 7.4 , s^2 66.20 , s 8.13

   ii. E 1.38 , s^2 3.172 , s 1.781

   iii. E 1.48 , s^2 4.044 , s 2.01
2. s^2 8.61 , s 2.93
3. $\bar{x}$ 21.5 , s^2 161 , s 12.7
4. s^2 1.168 , s 1.081
5. s^2 31.11 , s 5.58
6. E 1.35 , s^2 2.590 , s 1.609

REFERENCES

1. Bird, J (2017). Higher Engineering Mathematics (8^{th} Edition). New York, Ny: Routledge.
2. Cohn, P.M. (2017). Algebraic Numbers and Algebraic Functions. Taylor & Francis Ltd.
3. Chang Tze Hin, Ong Yoke Moo (2012). Additional Mathematics, Sasbadi.
4. https://www. Bumigemilang.com/pecutan-akhir-matematik-spm-soalan-kertas-2-jawapan/
5. https://afterschool.my/bm/berita/koleksi-soalan-spm-sebenar-tahun-2014- 2015-2016-2017-2018-spm-ulangan-
6. http://pjkweb2.pjk.edu.my/kspjk/
7. Larson. R 9^{th} Edition. (2011). Algebra & Trigonometry. Chicago State University:Pearson.
8. Novak, D. (1991). Basic Mathematics 3^{rd} ed. Massachusetts: DC Health and Company
9. Rehill, G.S. & McAiliffe, R. (2000). General Mathematics 3^{rd} ed. South Yarra:MacMillan Education Australia.
10. Sim, K.Y. Pua, K.T. & Chang, T.H. (2012), Additional Mathematics SPM, Sasbadi.

Terbitan :

POLITEKNIK MALAYSIA

e ISBN 978-967-2760-01-6

9 789672 760016